Pseudo-monoenergetic x-ray diffraction measurements using balanced filters for coherent-scatter computed tomography

S.R. Beath and I.A. Cunningham

Robarts Research Institute, Lawson Health Research Institute and Department of Medical Biophysics
The University of Western Ontario, 100 Perth Drive, London, Ontario, N6A 5K8, Canada

(Dated: October 21, 2008)

Coherent-scatter computed tomography (CSCT) is a method of producing tomographic images of the distribution of various tissue types that can be identified by their x-ray diffraction properties. Use of an x-ray tube degrades scatter-pattern angular resolution due to the x-ray spectral width, making it difficult to uniquely identify some materials. Two transmission filters with similar atomic numbers can be used to generate pseudo-monoenergetic scatter patterns. We describe this use of balanced filters (“Ross filters”) for CSCT, including an analysis of angular blurring mechanisms. It is shown, both theoretically and experimentally, that a relative RMS angular blur of approximately 3% can be achieved. Focal-spot size and beam width were the most important considerations determining angular blur after spectral width considerations.

PACS numbers: 87.57.Ce, 87.59.Fm, 87.59.-e, 07.85.Jy, 61.10.Nz, 61.10.Eq, 61.10.-i, 87.62.+n, 87.64.Bx
Keywords: coherent-scatter, x-ray diffraction, x-ray scattering, computed tomography, tissue composition, kidney stones, renal calculi, balanced filters, Ross filters

I. INTRODUCTION

X-ray coherent scatter (CS)\(^1\) gives rise to x-ray diffraction patterns that, even at diagnostic energies, can be used to distinguish some biological materials and provide tissue-composition information. This was shown initially by Bradley et al.,\(^2,3\) and Kosanetzky et al.,\(^4\) The team of Johns, Leclair, Wismayer, King and colleagues built on this work to determine how best to extract this information\(^5-9\) using both diffractometers\(^10\) and imaging detectors\(^11\) to measure this low-angle scatter from a number of materials.

Building on prior work by Harding and colleagues,\(^12-14\) we developed a coherent-scatter computed tomography (CSCT) facility in our laboratory\(^15,16\) to capture these CS patterns and generate tomographic images showing distributions of component materials. For example, minerals in urinary calculi have characteristic CS patterns that can be used for identification.\(^17\) This was used by Davidson, Batchelor and colleagues who generated tomographic images showing component-mineral distributions in intact stones.\(^18-20\)

A limitation to the use of (poly-energetic) x-ray tubes is the angular width of lines in diffraction patterns due to the x-ray spectral width.\(^11,21\) As a result, definitive identification of some materials can be difficult or impossible when mixed with unknown materials.

In 1926, Ross\(^22\) introduced the “balanced filter” technique as a method of obtaining nearly monoenergetic x-ray diffraction patterns. If two transmission filters are chosen such that their atomic numbers are similar, the difference pattern is similar to a monoenergetic pattern with energy equal to the average K-edge energy.

Application and optimization of balanced filters has since evolved. Efforts by Pierce\(^23\) involved finding balanced filter thicknesses empirically, as did Wollan,\(^24\) Boonstra\(^25\), and Gerrits and Bol.\(^26\) A method of correcting for filter imbalances with a third filter was explored by Kirkpatrick.\(^27,28\)

Soules et al.\(^29\) matched each filter’s transmission to suppress a \(K_\beta\) characteristic line outside of the narrow passband. Bol\(^30\) chose filter thicknesses to maximize the difference signal in the passband. Recently Saito\(^31\) used balanced filters for monoenergetic computed tomography. His method of filter optimization included choosing an optimal filter thickness ratio that matched transmission below the passband and then filter thicknesses to maximize differential transmission in the passband. Balanced filters were popular before the development of synchrotrons and they continue to be used in the study of x-ray physics,\(^32\) medical imaging,\(^33\) baggage inspection\(^34\) and elsewhere.

A relative spectral root-mean-square (RMS) width of 1-2% can be achieved by selecting two filters adjacent on the periodic table. However, since the method involves the difference of two measured diffraction patterns, noise in the difference pattern is often a consideration. Also, the width of Bragg peaks measured using balanced filters are increased by other considerations dependent on instrumentation and measurement geometry.

Noise can be reduced by choosing filters that have atomic number differences (\(\Delta Z\)) greater than unity at the expense of angular blur. The goal of this investigation is to examine all factors contributing to angular blur to enable appropriate filter selection and to apply this technique to coherent scatter imaging of mineral components found in kidney stones.
II. THEORY

A. Coherent scatter and scatter functions

The theoretical basis of coherent-scatter imaging has been described in detail elsewhere\textsuperscript{1,3,11,12,15} and only a brief summary is given here.

A thin “pencil” beam of x-rays is incident on the specimen and fully attenuated by a beamstop placed in front of an x-ray image intensifier (XRII) (Fig. 1). Low-angle scatter creates a two-dimensional scatter pattern incident on the detector. Many biological materials are either amorphous or polycrystalline, and coherent-scatter often shows circular symmetry similar to powder diffraction. The differential number of scattered photons from position \( l \) (per unit solid angle \( \Omega \) and path length through the specimen) that leave the specimen is given by:\textsuperscript{19}

\[
\frac{d^2S(l)}{d\Omega dl} \bigg|_{E,\theta} = N_I T(E) n_o(l) \frac{d\sigma(l)}{d\Omega} \bigg|_{E,\theta} \tag{1}
\]

where \( N_I \) is the number of incident photons, \( T(E) \) is the x-ray transmission factor through the specimen, \( n_o(l) \) is the volume density of electrons at \( l \) and \( d\sigma(l)/d\Omega \bigg|_{E,\theta} \) is the differential cross section per unit solid angle per electron, evaluated at x-ray energy \( E \) and scatter angle \( \theta \).\textsuperscript{10,36} At low scatter angles, the majority of scatter is due to coherent-scatter interactions. For a specimen of diameter \( W \), the differential number of scattered photons is given by:

\[
\frac{dN_S}{d\Omega} \bigg|_{E,\theta} = \int_W \frac{d^2S(l)}{d\Omega dl} \bigg|_{E,\theta} dl, \tag{2}
\]

and use of the x-ray spectrum \( dN_I(E)/dE \) gives:

\[
\frac{d^2S(E)}{d\Omega dE} \bigg|_{\theta} = \frac{dN_I(E)}{dE} T(E) \int_W n_o(l) \frac{d\sigma(l)}{d\Omega} \bigg|_{E,\theta} dl. \tag{3}
\]

When using an energy-integrating detector (such as the CsI-based XRII used in this study) to measure scatter patterns, the signal from a detector element at a position corresponding to scatter angle \( \theta \) is given by:

\[
d_\theta = k \Delta \Omega_p(\theta) \int_0^{kV} E_{abs}(E) \alpha(E) \frac{d^2S(E)}{d\Omega dE} \bigg|_{\theta} dE \tag{4}
\]

where \( k \) is the system gain (digital value per unit energy absorbed), \( \Delta \Omega_p(\theta) = \frac{a^2}{L^2} \cos^3 \theta \) is the solid angle of one detector element with area \( a^2 \), \( E_{abs}(E) \) is the energy absorbed (transferred to electron kinetic energy) locally in the detector material and is approximately equal to the incident photon energy less emitted characteristic-photon energy,\textsuperscript{36} and \( \alpha(E) \) is the detector quantum efficiency. Thus, the signal from a detector element at scatter angle \( \theta \) is:

\[
d_\theta = k \frac{a^2}{L^2} \cos^3 \theta \int_0^{kV} E_{abs}(E) \alpha(E) \frac{dN_I(E)}{dE} T(E) \frac{d\sigma(l)}{d\Omega} \bigg|_{E,\theta} dE dl. \tag{5}
\]

The line integral over \( l \) in Eq. (5) is of particular importance, but is made complicated by the dependence of many terms on x-ray energy. For the special case of \( N_o \) near-monoenergetic incident photons having energy \( E_o \), we obtain:

\[
d_\theta|_{E_o} = kN_o E_{abs}(E_o) \alpha(E_o) T(E_o) \frac{a^2}{L^2} \cos^3 \theta \int_0^{kV} n_o(l) \frac{d\sigma(l)}{d\Omega} \bigg|_{E_o,\theta} dE dl. \tag{6}
\]

The integrand is labeled the linear differential coherent-scatter coefficient, \( \gamma(l,\theta) \), equal to the probability per unit solid angle and path length (cm\(^{-1}\)) of a coherent-scatter photon scattering at angle \( \theta \):

\[
\gamma(l,\theta)|_{E_o} = n_o(l) \frac{d\sigma(l)}{d\Omega} \bigg|_{E_o,\theta}. \tag{7}
\]

The scaling factor \( k \) is generally not known and we work with estimates of \( \gamma(l,\theta) \), which we call the linear coherent-scatter function, \( C(l,\theta) \), defined as

\[
C(l,\theta) \equiv kN_o E_{abs}(E_o) \alpha(E_o) \gamma(l,\theta). \tag{8}
\]

The measured scatter signals \( d_\theta \) are therefore related to line integrals of the linear scatter function, \( C_{int}(\theta) \):

\[
C_{int}(\theta) \equiv kN_o E_{abs}(E_o) \alpha(E_o) \int_W \gamma(l,\theta)|_{E_o} dl = \frac{d_\theta}{\frac{a^2}{L^2} \cos^3 \theta T(E_o)}. \tag{9}
\]

This result shows that \( C_{int}(\theta) \) is proportional to line integrals of the differential scatter cross section \( d\sigma(l)/d\Omega \) and can be determined from measurements of \( d_\theta \) and \( T \). Transmission \( T \) is measured using the transmitted primary beam (with a detector placed on the beamstop in Fig. 1) as it has the same energy spectrum and essentially the same path as low-angle coherent scatter. \( C_{int}(\theta) \) is measured for a range of scatter angles \( \theta \) and acquired over a range of positions and azimuthal angles \( \phi \) in a plane through the specimen. This corresponds to first-generation CT geometry\textsuperscript{39} and filtered-backprojection reconstruction is used to determine \( C(\theta) \) values at each pixel location in a tomographic slice.
FIG. 2: (a) Radiation coming from a point source forming a beam of nominal width $B$ at the specimen will result in broadening of a Bragg peak at angle $\theta$ between $R_1$ and $R_2$ on the detector. Angles are exaggerated for clarity. (b) A Bragg peak at angle $\theta$ will be distributed between $R_1$ and $R_2$ due to the finite source size $S$.

B. Causes of scatter-function blur

Diffraction at angle $\theta$ (twice the Bragg angle) of x rays having wavelength $\lambda$ from crystal planes with spacings $d$ is governed by the Bragg relationship: $n\lambda = 2d\sin(\theta/2)$. Only first-order diffraction ($n = 1$) is considered here. Use of a broad spectral width as well as other considerations introduce a broadening of Bragg peaks corresponding to increased angular blur in C$(\theta)$. Starting with the simplest possible configuration, these mechanisms are described using approximate relationships to identify major factors contributing to angular blur. One-dimensional geometry is used for simplicity.

1. Geometric considerations

The specimen is irradiated with a small diverging beam having nominal width $B$ at the specimen, defined by a collimator placed a distance $d_1$ from the source, as illustrated in Fig. 2(a). Assuming a mono-energetic point source, a ray traveling along the beam top leaves the source at angle $\varphi$ from the central axis and is scattered by angle $\theta$ in the specimen, reaching the detector at position $R_1 = B/2 + L\tan(\varphi + \theta)$ where $\varphi$ is the beam divergence half-angle given by $\tan(\varphi) = B/2$. Using a small-angle approximation ($\varphi < 0.1^\circ$ and $\theta < 10^\circ$) gives $\tan(\varphi + \theta) \approx \tan \varphi + \tan \theta$ and thus scatter at $\theta$ may reach the detector anywhere over a range $\Delta R_B = R_1 - R_2 \approx 2(L + D)\tan \varphi = (L + D)B/D$, corresponding to a Bragg-peak RMS width of $\sigma_B$ where:

$$\sigma_B = \frac{1}{\sqrt{12}} \frac{\Delta R_B}{L} \approx \frac{1}{\sqrt{12}} \left( \frac{L + D}{LD} \right) B. \quad (10)$$

The $1/\sqrt{12}$ term results from determination of the RMS value of a rectangular distribution.

Focal-spot size $S$ also introduces blur in $\theta$ as illustrated in Fig. 2(b) where a ray coming from the bottom of the focal spot and passing through the center of the collimator opening is scattered in the specimen by $\theta$ and incident on the detector at $R_1 = (D - d_1)\tan \phi + L\tan(\varphi + \theta)$ where $\phi$ is the source half-angle given by $\tan \phi = S/4d_1$. A ray coming from elsewhere on the focal spot will result in the same Bragg peak displaced by an amount that depends only on the source distance from the central axis and other Thus, scatter at $\theta$ is blurred over a distance $\Delta R_S \approx (L + D - d_1)S/d_1$ on the detector, resulting in a Bragg-peak RMS width $\sigma_S$ where:

$$\sigma_S \approx \frac{1}{\sqrt{12}} \frac{(L + D - d_1)}{Ld_1} S. \quad (11)$$

By using the small-angle approximation and choosing the paths to converge at the collimator position in this calculation, focal-spot blur as described acts independently of, and in addition to, beam-width blur.

Large specimens introduce a rectangular broadening of Bragg peaks due to variations in the scatter-event position along the path $l$. For a specimen of diameter $W$, scatter may be incident on the detector over a range $\Delta R_W = W\tan \theta$, corresponding to an RMS width $\sigma_W$ given by:

$$\sigma_W = \frac{1}{\sqrt{12}} \frac{W\tan \theta}{L}. \quad (12)$$

Spatial resolution of the imaging detector results in a small additional blur of the scatter patterns. The system line-spread function was measured (slanted-edge method) and shown to be approximately Gaussian with an RMS width of $\tau = 0.24$ mm (including the effect of pixel size), giving:

$$\sigma_{\tau} \approx \frac{\tau}{L}. \quad (13)$$

Scatter functions are calculated by integrating $d_{i,j}$ in concentric rings of nominal width one pixel $x_o$, normalized by solid angle. However, the rings are centered on the observed scatter patterns which do not normally align with detector pixels. Linear interpolation of $d_{i,j}$ to the nearest ring introduces an additional rectangular blur by approximately $2x_o$, corresponding to an increase in Bragg-peak RMS angular width of $\sigma_R$, where:

$$\sigma_R \approx \frac{1}{\sqrt{12}} \frac{2x_o}{L}. \quad (14)$$
2. Coherence-length considerations

At diagnostic x-ray energies, multiple x-ray interactions are highly unlikely and the kinematical approximation is used.\cite{54} Both the width and height of Bragg peaks depend on the number of lattice atoms contributing to interference patterns. This is determined by crystal size for small crystals when using a highly coherent beam,\cite{54} and potentially by curvature of the spherical wavefront. A single photon may occur from scattering centers over a region of up to dimension $L_c$ for which the wavefront is approximately in phase. In Fig. 3, this requires $\delta$ to be a small fraction of $\lambda$. Thus, choosing $\delta = 4\lambda$ gives the condition $\delta = D(1-\cos \vartheta) = \lambda/4$. Since $\sin \vartheta = \frac{L_c/2}{D} \approx \vartheta$, we obtain

$$L_c \approx 2D \arccos \left(1 - \frac{\lambda}{4D}\right). \quad (15)$$

Large $L_c$ values result in strong sharp peaks in the scatter functions with the peak full-width at half-maximum (FWHM) given by the Scherrer formula:\cite{54}

$$\text{FWHM}_C \approx \frac{0.9\lambda}{L_c \cos \theta}. \quad (16)$$

For small $\theta$, peak shape is approximately proportional to $\text{sinc}^2 \theta$.\cite{54} Although the RMS width of a squared sinc function is undefined, the RMS width $\sigma$ of a Gaussian peak is related to its FWHM by $\sigma = \frac{1}{\sqrt{8\ln 2}} \times \text{FWHM}$. Approximating the central region of $\text{sinc}^2 \theta$ as a Gaussian curve having the same FWHM and height gives the Bragg peak RMS width as:

$$\sigma_C \approx \frac{1}{\sqrt{8\ln 2}} \times \frac{0.9\lambda}{L_c \cos \theta}. \quad (17)$$

3. Spectral-width considerations

Blur caused by the spectral width, assuming a relatively narrow spectrum, is obtained by writing the Bragg relationship as $\sin(\theta/2) = \lambda/2d = hc/2Ed$ and differentiating with respect to $E$. For a rectangular spectral width $\Delta E$, the Bragg-peak RMS width is given by:

$$\sigma_E \approx \frac{2\Delta E \sin(\theta/2)}{\sqrt{12E} \sqrt{1 - 4\sin^2(\theta/2)}}. \quad (18)$$

4. Total scatter-function blur

Each blurring mechanism described above results in additional broadening of Bragg peaks. While blur is a function of $\theta$ for some mechanisms, the extent of each blur for a specified Bragg peak can be approximated as being constant over the width of the blur, and thus each blur can be represented as a convolution integral for specified $\theta$. Since the RMS width of a line blurred by a cascade of convolutions is the quadrature sum of the RMS width of each blurring kernel, the expected Bragg-peak RMS width in the measured scatter functions is given by $\sigma_\theta$ where:

$$\sigma_\theta^2 \approx \sigma_B^2 + \sigma_S^2 + \sigma_W^2 + \sigma_r^2 + \sigma_\rho^2 + \sigma_L^2 + \sigma_E^2. \quad (19)$$

It should be noted that both $\sigma_W$ and $\sigma_E$ increase with $\theta$, resulting in Bragg peaks being broadened more on the high-$\theta$ side and therefore an increase in the peak centroid position. This effect was ignored in the above theoretical description and suggests that measured peaks may be shifted slightly toward larger $\theta$ values than predicted.

C. Pseudo-monoenergetic scatter functions using balanced filters

The balanced-filter method utilizes two elemental transmission filters, having similar K-edge energies, placed near the source. The spectrum transmitted through the low-Z filter is given by:

$$\frac{dN_L(E)}{dE} = \frac{dN_0(E)}{dE} e^{-\left(\frac{\mu}{\rho}\right)_L(E) \rhoLt_L}, \quad (20)$$

where $(\frac{\mu}{\rho})_L$, $\rho_L$ and $t_L$ are the mass attenuation coefficient, mass density and thickness of the low-Z filter. A similar equation can be written for the high-Z filter. Scatter functions obtained with each filter are represented as $C_H(\theta)$ and $C_L(\theta)$, and the desired monoenergetic scatter function, $C(\theta)$, is given by:

$$C(\theta) \approx C_\Delta(\theta) = C_H(\theta) - C_L(\theta). \quad (21)$$

1. Optimal filter design

Filter thicknesses are chosen to match transmission at an energy immediately outside and below the passband,
The effective energy of the difference spectrum is determined by the average filter K-edge energy. Potential filters are summarized in Table I.

3. Factors affecting scatter-function blur

The impact on angular blur as determined by Eq. (19) of various source-to-object (D) and object-to-detector (L) distances using Er-Tm filters is summarized in Fig. 6. For the beam and focal-spot sizes used, D has an optimal value with a weak L dependence where blur is minimized. In general, D should be minimized to reduce inverse-square losses but not less than 25 or 30 cm. Increasing L always improves angular blur, but not strongly. Dimensions used in this study, unless noted otherwise, are D = 41.5 cm and L = 39.5 cm - convenient values chosen close to the optimal for blur reduction.

III. MATERIALS AND METHODS

A. Coherent-scatter system description

The CSCT system consists of a tungsten anode x-ray tube (Varian Medical Systems, Palo Alto, CA) and CsI XRII (Precise Optics, Bay Shore, NY) mounted on an optical bench using 1st-generation CT geometry. A 1-mm-square x-ray beam is filtered by Tm and Er filters (ESPI Metals, Ashland, OR) located close to the tube. Both filters were 0.0127 cm thick as this is very close to the optimal thickness. The beam interrogates the specimen placed on a translate-rotate-elevate motorized stage. Transmitted photons are incident on a Lanex-regular screen (Kodak / Carestream, Rochester, NY) coupled to a photodiode and mounted on a beam stop. Scattered photons (θ < 15°) are incident on the XRII which is coupled to a CCD FireWire camera (Prosilica, Burnaby, BC).

Images of scatter patterns were obtained after dark subtraction and correction for XRII geometric distortion. Values of di,j were determined by averaging di,j values in concentric rings about the center. The integral scatter function, Cint(θ), was determined using Eq. (9). Focal-spot size (FWHM) was measured using a pin-hole camera to be 1.2 x 0.9 mm, consistent with the NEMA specification of 0.9 x 0.6 mm.

B. Validation of scatter-function-blur model

The theoretical model of scatter-function blur using balanced Er-Tm filters in Eq. (19) was validated using 0.5 cm and 1.5-cm diameter cylinders of aluminum (Al) powder (VWR International, West Chester, PA). Aluminum has a face-centered cubic structure (Fm-3m, space-structure 225), and cell length a = 4.0497 Å (App 5 of Cullity). Angles of the first five Bragg peaks were determined for Al powder at 58.4 keV using commercial software (Crystal Impact) as illustrated in Fig. 7.

C. Pseudo-monoenergetic scatter functions

Scatter patterns and functions from commercial powdered preparations of four representative kidney-stone
FIG. 5: (a) Theoretical calculation of 120 kV x-ray spectra transmitted through Tm (Z=69) and Er (Z=68) filters using thicknesses from Table I. (b) Difference spectrum, \( \Delta N(E) \), with \( E_K \approx 58.4 \text{ keV} \) and passband width \( \Delta E_K \approx 1.9 \text{ keV} \).

![Graph showing x-ray energy and transmitted spectra](image)

![Graph showing difference spectrum](image)

**FIG. 6:** Total RMS angular blur in degrees at \( \theta = 2^\circ \) from the test Ca stone using balanced Er-Tm filters for various source-to-object (D) and object-to-detector (L) distances (\( x_o = 0.28 \text{ mm}, \tau = 0.24 \text{ mm}, B = 0.1 \text{ cm}, 120 \text{ kV}, W=1 \text{ cm} \)).

![Graph showing total RMS angular blur](image)

**TABLE I:** Possible balanced-filter combinations using Eqs. (22) and (23). Mean effective energy in the difference spectrum is \( \bar{E} \), the average K-edge energy. Spectral width is determined by \( \Delta E_K \), the difference in K-edge energies.

<table>
<thead>
<tr>
<th>Low-Z Filter</th>
<th>( t_L ) [cm]</th>
<th>High-Z Filter</th>
<th>( t_H ) [cm]</th>
<th>( \bar{E} ) [keV]</th>
<th>( \Delta E_K ) [%]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>57 - Lanthanum</td>
<td>0.013</td>
<td>58 - Cerium</td>
<td>0.010</td>
<td>39.7</td>
<td>3.8</td>
<td>La oxidizes in air</td>
</tr>
<tr>
<td>61 - Promethium</td>
<td>0.016</td>
<td>64 - Gadolinium</td>
<td>0.013</td>
<td>47.7</td>
<td>10.6</td>
<td>Pr Radioactive</td>
</tr>
<tr>
<td>64 - Gadolinium</td>
<td>0.015</td>
<td>66 - Dysprosium</td>
<td>0.013</td>
<td>52.0</td>
<td>6.8</td>
<td></td>
</tr>
<tr>
<td>67 - Holmium</td>
<td>0.016</td>
<td>69 - Thulium</td>
<td>0.014</td>
<td>57.5</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>68 - Erbium</td>
<td>0.015</td>
<td>69 - Thulium</td>
<td>0.013</td>
<td>58.4</td>
<td>3.3</td>
<td>W anode K( \alpha ) x-rays</td>
</tr>
<tr>
<td>72 - Hafnium</td>
<td>0.013</td>
<td>73 - Tantalum</td>
<td>0.009</td>
<td>66.4</td>
<td>3.1</td>
<td>W anode K( \beta ) x-rays</td>
</tr>
</tbody>
</table>

**IV. RESULTS**

**A. Scatter-function angular blur**

Figure 7(a) shows a pseudo-monoenergetic scatter pattern from Al powder obtained using balanced Er-Tm filters. Figure 7(b) shows a comparison of measured \( C_{int}(\theta) \) with the least-squares-fit curve and the expected positions of the first five Bragg peaks (58.4 keV). Excellent agreement was observed after stretching the theoretical angular scale by 4%, likely required due to asymmetric blur (see Sec.II B 4) and/or imprecise knowledge of true specimen-detector distance \( L \). Peaks D and E showed significant overlap and were fit to a single Gaussian curve.

The factors affecting angular blur in Eq. (19) are summarized in Fig. 8. At low scatter angles, focal-spot size and beam width are the limiting factors. Both x-ray spectral width and specimen size adds blur that is proportional to \( \theta \) and have an effect similar to focal spot and beam width at large angles. These results are specific to the test geometry indicated. For Er-Tm filters, the important Bragg peaks from kidney-stone components are between 1 and 5\( ^\circ \) where angular dependence is modest.

Minerals (calcium oxalate monohydrate (COM), calcium phosphate dihydrate (CPD), cystine (CYS), magnesium ammonium phosphate hexahydrate (MAP, struvite), Sigma-Aldrich Co. and Fluka Chemika), were obtained using balanced Er-Tm filters (120 kV, 300 mAs, \( D = 41.5 \text{ cm}, L = 39.5 \text{ cm}, W = 1 \text{ cm} \)) to demonstrate the extent to which scatter-function peaks can be resolved. Results were compared with copper K\( \alpha \) x-ray diffractometry (XRD) at \( \sim 8 \text{ keV} \).
FIG. 7: (a) Powdered-aluminum scatter pattern obtained using balanced Er-Tm filters (140 kV, 192 mAs, \( D = 63 \) cm, \( L = 55 \) cm, \( W = 1.5 \) cm). (b) Comparison of corresponding integral scatter function with least-squares-fit curve consisting of four Gaussian peaks. Also shown are positions of the first five theoretical peaks from Al powder at 58.4 keV.

FIG. 8: Measured Bragg-peak RMS width for Al powder compared to the theoretical prediction given by Eq. (19) and its components (Er-Tm filters, \( E = 58.4 \) keV, 140 kV, 192 mAs, \( D = 63 \) cm, \( L = 55 \) cm, \( W = 1.5 \) cm). Under these conditions, the scatter angles 1 - 6° are most important for identifying kidney-stone minerals.

The measured RMS width of peaks A, B and C from powdered Al are also shown in Fig. 8. Measured blur is comparable to predicted blur for peaks A and B. Peak C at 8° showed a blur approximately 50% greater than predicted. While Eq. (19) is an approximation and complete agreement is not expected, we believe additional factors are important at larger \( \theta \) such as the effects of oblique x-ray incidence, curvature of the XRII input phosphor, or off-axis electron focussing in the XRII.

Figure 9 shows measured and predicted RMS scatter-function blur (peak A) for various object-to-detector distances \( L \) \((W = 0.5 \) and 1.5 cm Al cylinders). Increasing \( L \) decreases angular blur, whereas increasing \( W \) increases angular blur. Similar to Fig. 8, measured peak widths show more blur than predicted but confirm the expected trends.

B. Pseudo-monoenergetic scatter functions from kidney-stone minerals

Figure 10 shows a comparison of scatter patterns from common kidney-stone minerals obtained using XRD and balanced Er-Tm filters. While the XRD results have superior angular resolution, the balanced-filter results also show distinct rings. The XRD results required 20-min exposures while the balanced-filter results required exposures of a few seconds. Integral scatter functions from data in Fig. 10 are shown in Fig. 11.

V. CONCLUSION

It has been shown, both theoretically and experimentally, that an effective relative spectral RMS width of approximately 3% can be achieved using a diagnostic x-ray tube and balanced filters.
FIG. 9: Measured Bragg-peak RMS width for Al powder (peak A) at \( L = 25 \) and 55 cm compared to the theoretical prediction given by Eq. (19) for \( W = 0.5 \) and 1.5 cm (Er-Tm filters, 140 kV, 192 mAs, \( D = 63 \) cm). Results for \( L = 55 \) cm also show measurement imprecision (standard deviation in seven trials).

The theoretical model of scatter-function blur was validated by experimental measurements for Al powder at 58.4 keV. Measured width agreed with theory below approximately \( 6^\circ \). It is thought that oblique incidence on the XRII curved input phosphor and off-axis electron-focussing issues may be responsible for the increased blur at higher angles.

As a generalization, the following additional observations are made from this work:

1. Focal-spot size and beam width are the primary considerations limiting angular blur in scatter functions. An RMS width of \( \sim 2\% \) (Fig. 8) can be achieved.

2. The source-specimen distance should be as small as possible to maximize scatter intensity, but not less than 25 or 30 cm for a 1-mm focal spot and beam (Fig. 6). Blur is reduced only slowly with increasing specimen-detector distance.

3. For the conditions in this study, filter atomic number differences of two or three (Table I) will maximize difference-spectrum intensity with only a minor increase in angular blur.

Acknowledgments

The authors are grateful to the Canadian Institutes of Health Research and the Lawson Health Research Institute for financial support. The assistance of Drs. Sergey Lazarev and Melanie Davidson is gratefully acknowledged.

FIG. 11: Er-Tm scatter functions (120 kV, 300 mAs, $W = 1$ cm, $D = 42$ cm, $L = 40$ cm) from common kidney stone minerals.

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