Normalization of the modulation transfer function: The open-field approach

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The modulation transfer function (MTF) is widely used to describe the spatial resolution of x-ray imaging systems. The MTF is defined to have a zero-frequency value of unity, and it is common practice to ensure this by normalizing a measured MTF curve by the zero-frequency value. However, truncation of the line spread function (LSF) within a finite region of interest (ROI) results in spectral leakage and causes a reduction in the measured MTF zero-frequency value equal to the area of truncated LSF tails. Subsequent normalization by this value may result in inflated MTF values. We show that open-field normalization with the edge method produces accurate MTF values at all nonzero frequencies without need for further normalization by the zero-frequency value, regardless of ROI size. While both normalization techniques are equivalent for a sufficiently large ROI, a 5% inflation in MTF values was observed for a CsI-based flat-panel system when using a 10 cm ROI. Use of open-field normalization avoids potential inflation caused by zero-frequency normalization. © 2008 American Association of Physicists in Medicine.

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I. INTRODUCTION

The modulation transfer function (MTF) is a Fourier metric widely used to describe spatial resolution of linear and shift-invariant x-ray imaging systems.1–3 The one-dimensional MTF is equal to the modulus of the Fourier transform of the system line-spread function (LSF),4 and it is generally accepted that the LSF be defined as having unity area when integrated over all space, implying (through the central ordinate theorem)5 that the MTF will have a unity zero-frequency value.6 For this reason, many investigators force the zero-frequency value to unity by scaling the MTF at all frequencies,7,6–13 and this is also the recommendation of an International Electrotechnical Commission (IEC) document describing methods for measuring and calculating the MTF.14

Normalization by the zero-frequency value is correct as long as the entire LSF—including tails—is measured. However, the LSF is measured using a finite image region of interest (ROI) and the LSF of many systems have tails that extend beyond the ROI. As a result, the area of the LSF that falls within the ROI may be less than unity with the consequence that forcing the MTF zero-frequency value to unity may be incorrectly inflating MTF values at all spatial frequencies.

Throughout the last five decades, several investigators have noted the effect of limited ROI size on MTF measurements. In 1964, Rossmann15 showed that truncation of extended tails in the LSF of a radiographic screen-film system resulted in inflated MTF values and recommended that the LSF not be truncated until it reduces to 1% of the peak value in the Fourier integral. More recently, Carton et al.11 showed that the size of a slanted edge (and ROI) has a small overall effect on calculated MTF curves but a notable effect on the measured low-frequency drop (LFD) of a commercial storage-phosphor-based digital mammography system, and recommended use of an edge device of at least 5 cm in size. Using simulated data, Samei and colleagues16 showed that a small ROI inflates MTF values. Gopal and Samant13 investigated the use of bar-pattern MTF techniques on megavoltage imaging and concluded that an ROI of 10 cm is required to accurately determine the zero-frequency value. The IEC recommendation14 is to use a 16 cm x-ray beam but is less specific on the ROI size. Excessive truncation of the LSF can also introduce oscillations in the MTF.17,18

The problem stems from inadequate assessment of the zero-frequency component due to incomplete capture of the LSF tails in the limited-size ROI. Truncation of the tails results in a reduced zero-frequency component of the measured transfer function given by the Fourier transform of the LSF. Subsequent normalization to unity incorrectly inflates the calculated MTF.15

Calculations of the detective quantum efficiency (DQE) include a squared-MTF term and, hence, errors in the MTF result in almost twice that error in the DQE. In the context of standardized DQE-measurement protocols and comparisons,
it is important that these potential errors be well understood. In addition, metrics that depend on the MTF-squared integral (such as Wagner’s bandwidth integral or recent work on temporal MTF properties of fluoroscopic systems are very sensitive to systematic biases and normalization errors in the MTF.

We believe that one should expect the measured MTF zero-frequency value to be less than unity because the LSF will always be truncated to some extent. In this article, we show that with the use of an open-field image, improved normalization of the MTF can be achieved even when LSF tails are truncated. “Open-field normalization” has been used previously in MTF measurements, and is sometimes used simply to account for gain differences between detector elements. We show that open-field normalization results in a measured MTF that avoids inflation caused by the truncation of the LSF tails and subsequent normalization by the zero-frequency value.

II. THEORY

Normalization issues affect all methods used to measure the MTF. We describe linear-systems models for both edge and slit-based methods. It is assumed that both the mean x-ray distribution incident on the measurement apparatus, \( \bar{q}(x) \), and the detector have infinite spatial extent. Thus, while a finite ROI is described, it is assumed the ROI is small compared to the detector size and detector edge effects are negligible. It is also assumed that while aliasing issues in digital systems may affect the edge image, they do not affect the open-field image. This is not a problem as we will also assume the incident x-ray field changes slowly over the detector area. Results are described using one-dimensional geometry for simplicity, but extension to additional dimensions is straightforward.

II.A. Edge method

II.A.1. Open-field normalization

Similar to previous descriptions of the edge technique for measuring the MTF, we consider a semi-infinite sheet of radiopaque material with a precision-machined edge and describe the transmitted x-ray distribution incident on the detector as \( \bar{q}(x)H(x) \) where \( H(x) \) is the Heaviside function (a value of 0 corresponds to presence of edge material). The measured presampling edge profile obtained using the slanted-edge method, \( E(x) \), is given by:

\[
E(x) = k\bar{q}(x)H(x) \ast \text{lsf}(x),
\]

where \( k \) is a system gain factor, \( \text{lsf}(x) \) is the true system LSF corresponding to a detector with infinite extent, and

\[
H(x) = \begin{cases} 
1 & x > 0 \\
0 & \text{otherwise}.
\end{cases}
\]

An open-field image is obtained without the edge material. If the edge image is first normalized by this open-field image, the open-field-normalized edge profile, \( E^{\text{of}}(x) \), is given by:

\[
E^{\text{of}}(x) = \frac{k\bar{q}(x)H(x) \ast \text{lsf}(x)}{k\bar{q}(x) \ast \text{lsf}(x)}.
\]

Data from an ROI of extent \( L_x \) centered on the edge are identified using a rectangular function \( \Pi \left( \frac{x}{L_x} \right) \) (with \( \Pi \left( \frac{x}{L_x} \right) \big|_{x=1} = 1 \)). Our estimate of the open-field-normalized LSF, \( \text{lsf}^{\text{of}}(x) \), is obtained by taking the derivative with respect to \( x \) within the ROI:

\[
\text{lsf}^{\text{of}}(x) = \frac{d}{dx} \left[ E^{\text{of}}(x) \Pi \left( \frac{x}{L_x} \right) \right]_{|x|<L_x/2}
\]

\[
= \frac{q(0)\text{lsf}(x)}{\bar{q}(x) \ast \text{lsf}(x)} [\bar{q}'(x)H(x) \ast \text{lsf}(x) + q(x)H(x) \ast \text{lsf}(x)] \Pi \left( \frac{x}{L_x} \right) \]

\[
- \frac{[\bar{q}(x)H(x) \ast \text{lsf}(x)][\bar{q}'(x) \ast \text{lsf}(x)] \Pi \left( \frac{x}{L_x} \right)}{[\bar{q}(x) \ast \text{lsf}(x)]^2},
\]

where prime denotes a derivative. While this result is only evaluated within the ROI, we do not explicitly show the condition \( |x|<L_x/2 \) after the first line and in the remainder for simplicity.

The profile is normally slowly changing in \( x \) and, hence, \( \bar{q}(x)\text{lsf}(x) = \bar{q}(x) \) and \( q'(x)\text{lsf}(x) = \bar{q}'(x) \), giving:

\[
\text{lsf}^{\text{of}}(x) = \frac{\bar{q}(0)}{\bar{q}(x)}\text{lsf}(x) \Pi \left( \frac{x}{L_x} \right) + \frac{\bar{q}'(x)H(x) \ast \text{lsf}(x)}{\bar{q}(x)}
\]

\[
- \frac{[\bar{q}(x)H(x) \ast \text{lsf}(x)][\bar{q}'(x) \ast \text{lsf}(x)] \Pi \left( \frac{x}{L_x} \right)}{[\bar{q}(x) \ast \text{lsf}(x)]^2}.
\]

The term in square brackets is a consequence of a nonuniform \( \bar{q}(x) \) in the open-field x-ray profile, resulting from the heel effect or other considerations. It is shown in Appendix A that under the condition

\[
L_x < \frac{\bar{q}(0)}{\max[q'(x)]_{|x|<L_x/2}},
\]

where \( \max(\cdot) \) determines the maximum value over the ROI, this term can be ignored giving:

\[
\text{lsf}^{\text{of}}(x) = \bar{q}(0)\text{lsf}(x) \Pi \left( \frac{x}{L_x} \right).
\]

The term \( \bar{q}(0)/\bar{q}(x) \) is a window-like scaling resulting from nonuniformity in the incident x-ray distribution.

If the chosen ROI size is not an integer multiple of any nonnegligible input signal component wavelengths, the calculated Fourier spectrum will contain additional frequency components, a phenomenon known as spectral leakage. This can be described as convolution with a sinc function in the Fourier domain. Some investigators use a window function, \( w(x) \) (e.g., Hann window), to reduce spectral leakage.
If an LSF has long tails, spectral leakage will necessarily be present, as the ROI will be smaller than some of the input signal components.

The Fourier transform of Eq. (7) with the inclusion of a normalized window function \( w(x) \) is:

\[
\mathcal{T}^\text{sf}(u) = B(u) \ast T(u) \ast L_x \ast \text{sinc}(L_x u) \ast W(u)
\]  
(8)

where \( B(u) \) and \( W(u) \) are the Fourier transforms of \( \tilde{q}(0)/\tilde{q}(x) \) and \( w(x) \), respectively, \( T(u) \) is the true system transfer function, and \( \text{sinc}(L_x u) = \sin(\pi L_x u)/\pi L_x u \). In practice, beam nonuniformities are generally small and \( B(u) \) can be ignored, giving the MTF determined using the open-field-normalized edge method, \( \mathcal{MTF}^\text{sf}(u) \), as:

\[
\mathcal{MTF}^\text{sf}(u) = \left| \mathcal{T}^\text{sf}(u) \right| = \left| T(u) \ast L_x \ast \text{sinc}(L_x u) \ast W(u) \right|.
\]  
(9)

The open-field-normalized MTF differs from the true system MTF, \( |T(u)| \), by convolution with \( L_x \ast \text{sinc}(L_x u) \) and \( W(u) \). The convolution with the sinc function introduces spectral leakage, as illustrated in Fig. 1. The window function reduces the effect of the oscillations in the frequency domain caused by spectral leakage, at the expense of additional blur. Thus, if there is a low-frequency drop in the true system transfer function, \( T(u) \), both leakage (as described by convolution with the sinc function) and blur due to a window function if used, will broaden the peak in the MTF near the zero frequency and decrease the zero-frequency value. Subsequent normalization by the zero-frequency value will inflate the MTF at all other frequencies. In areas of the MTF that are slowly changing—generally the case for all but very low frequencies—leakage and windows have minimal effect. The conclusion from this calculation is that open-field normalization gives an unbiased estimate of the true MTF at all frequencies except close to zero, where the measured MTF may be less than the true MTF.

This result describes the impact of using a (possibly windowed) LSF measured in a finite ROI, with the measured zero-frequency value being equal to the truncated area of the LSF (central-ordinate theorem\(^5\)), \( A \), with \( A < 1 \):

\[
\mathcal{MTF}^\text{sf}(0) = \int_{-L_x/2}^{L_x/2} \text{lsf}(x)w(x)dx = A,
\]  
(10)
as illustrated in Fig. 1. It also shows that by normalizing the measured edge profile with an open-field image, accurate MTF values are obtained everywhere except near zero frequency when in the presence of a low-frequency drop.

II.A.2. Zero-frequency normalization

The effect of normalizing the MTF by the zero-frequency value is described using a similar model. The edge profile \( \text{E}(x) \) from Eq. (1) is differentiated directly while requiring \( \tilde{q}(0)/\tilde{q}(x) \) and \( q(x) \) over the ROI. Taking the Fourier transform of the result gives:

\[
\mathcal{T}^\text{sf}(u) = k\tilde{q}(0)T(u) \ast L_x \ast \text{sinc}(L_x u) \ast W(u).
\]  
(11)

Taking the modulus and normalizing gives:

\[
\mathcal{MTF}^\text{sf}(u) = \frac{\left| \mathcal{T}^\text{sf}(u) \right|}{\left| \mathcal{T}^\text{sf}(0) \right|} = \frac{1}{A} \left| T(u) \ast L_x \ast \text{sinc}(L_x u) \ast W(u) \right|
\]  
(12)

Truncation of LSF tails results in \( A < 1 \) and, thus, normalization at zero frequency overstates the MTF at nonzero frequencies by the factor \( 1/A \).

II.B. Slit method

A derivation of the results obtained using the slit method with open-field normalization is shown in Appendix B. If the slit width \( a \) is known, the MTF can be corrected for finite slit width by dividing by \( |a \ast \text{sinc}(au)| \), giving:

\[
\mathcal{MTF}^\text{sf}_{\text{slit}}(u) = \frac{\left| [a \ast \text{sinc}(au)]T(u) \ast L_x \ast \text{sinc}(L_x u) \ast W(u) \right|}{\left| a \ast \text{sinc}(au) \right|} = \mathcal{MTF}^\text{sf}(u).
\]  
(13)

Zero-frequency normalization results in:

\[
\mathcal{MTF}^\text{sf}_{\text{slit}}(u) = \frac{\mathcal{MTF}^\text{sf}_{\text{slit}}(u)}{\mathcal{MTF}^\text{sf}_{\text{slit}}(0)} = \frac{1}{A} \mathcal{MTF}^\text{sf}_{\text{slit}}(u).
\]  
(14)

This result shows that, similar to the edge method, zero-frequency normalization with the slit method inflates MTF values when in the presence of a low-frequency drop. Slit width may not be known accurately, and affects the results of both normalization techniques.
III. METHODS

A comparison of open-field and zero-frequency normalization techniques was performed using both simulated and experimental data. In all cases, the presampling LSF was determined using the slanted-edge technique with an over-sampling factor of eight and application of a Hann window function\(^{27}\) (also commonly referred to as a Hanning window\(^ {28}\)). The size of the ROI used to calculate the LSF was varied between \(L_x=1\) and 15 cm across the edge (centered on the edge), and \(L_y=5\) and 10 cm in the perpendicular direction.

III.A. Simulation validation

Simulated images from a 20 \(\times\) 20 cm detector with square pixels of width \(a=0.2\) mm were generated of an ideal edge slanted by 1.5° with respect to the y axis. Spatial blur was added by convolving with a kernel consisting of the sum of two Gaussian curves having standard deviation widths of \(\sigma_1=5.0\) mm and \(\sigma_2=50.0\) mm, respectively, and normalized such that the simulation MTF curve was given by:

\[
\text{MTF}_w\left(u \right) = \left[0.7e^{-2(\pi \sigma_1 u)^2} + 0.3e^{-2(\pi \sigma_2 u)^2}\right] \text{sinc}(au).
\]

This corresponds to an MTF with a low-frequency drop of 0.3. While this is larger than typical, it presents a severe test of the normalization methods.

III.B. Clinical-system analysis

The MTF of a General Electric Co. Revolution XQ/i CsI-based flat-panel detector in clinical service was determined using a precision-machined tungsten edge device slanted at \(\sim 5^\circ\) from the y axis and placed in the center of the detector with a 16.5 \(\times\) 16.5 cm exposed detector area using an RQA-5 (70 kV) beam. The detector cover was left in place and the scatter-rejection grid removed. The incident exposure was 0.6 mR per image and the image-plane pixel size was 0.20 \(\times\) 0.20 mm.

A separate monitor detector was used to confirm there were no variations in x-ray tube output between open-field image and edge image acquisitions.

IV. RESULTS

IV.A. Simulation results

Figure 2 shows a comparison of MTF curves determined from the simulation images corresponding to ROI sizes of \(L_x=1\) and 10 cm across the edge with \(L_y=5\) cm. The theoretical MTF based on Eq. (15) is labeled “simulation truth.” Symbols on the two 1 cm curves identify the MTF sample points determined by the fast Fourier transform (FFT). The 1 cm open-field MTF (labeled “of”) has a zero-frequency value of \(A=0.70\) as expected due to the theoretical LFD of 0.30, and shows reasonable agreement with simulation truth at all nonzero frequencies. The zero-frequency-normalized result shows inflation at nonzero frequencies by \((1-A)/A=43\%\).

For the \(L_x=10\) cm ROI, the number of sample points has increased by a factor of 10, and both methods show improved agreement with simulation truth. The open-field-normalized result has a zero-frequency value of 0.97 and the zero-frequency-normalized result is inflated by 3\% at nonzero frequencies. A comparison between open-field-normalized zero-frequency values and calculated area of the truncated windowed LSF is shown in Table I. Excellent agreement is observed, as predicted by Eq. (10).

IV.B. Clinical-system results

Calculated MTF curves for the clinical system were determined for ROI sizes between \(L_x=1\) and 15 cm. Results for 1 and 10 cm are displayed in Fig. 3, and more noise is observed for the 10 cm ROI results, as is expected for a larger ROI. Open-field normalization gave consistent results at nonzero frequencies for all ROI sizes tested as predicted by Eq. (9). The zero-frequency values are listed in Table II, showing that 12\% of the LSF area was truncated by the 1 cm
ROI, and 5% by the 10 cm ROI. As a result, use of zero-frequency normalization inflates MTF values at nonzero frequencies by the fraction

$$\frac{\text{MTF}^{\text{of}}(0) - \text{MTF}^{\text{zf}}(0)}{\text{MTF}^{\text{of}}(0)} = \frac{1 - A}{A},$$

as summarized in Table II and illustrated in Fig. 3.

The requirement from Eq. (5) that the term in brackets be small compared to the LSF value was satisfied for the clinical system over the ROI. No significant differences were found between MTF values calculated using ROIs of $L_x = 10$ cm and 5 cm, as determined using a $\chi^2$ test.

**V. DISCUSSION**

The theoretical description of the edge method presented here identified restrictions on the maximum ROI size in Eq. (7) for open-field normalization and in Eq. (11) for zero-frequency normalization techniques. These conditions give practical limits as larger values of $L_x$ may introduce errors to the calculated MTF due to beam nonuniformities, such as the heel effect. Excellent agreement was found between the open-field-normalized MTF results on the clinical system indicating the condition was satisfied for $1 \text{ cm} \leq L_x \leq 15$ cm. While we believe these conditions have not been previously reported, they are not unique to our technique and are required for all edge methods. A window-like term, $\bar{q}(0)/\bar{q}(x)$, was also noted for open-field normalization resulting from beam nonuniformities. While it could be corrected if the incident x-ray profile could be measured, this would require disabling any “beam-flattening” corrections that may be applied by the system. However, this effect is very small if the x-ray profile is relatively uniform, as normally achieved in practice.

Only one-dimensional MTF results are shown, corresponding to integration parallel to the edge over $-\infty < y < \infty$. This is inherent to the slanted-edge method, but requires that the ROI y size, $L_y$, be large compared to the resolving distance in the y direction. A generalization of the theory described here shows that y direction truncation also reduces the measured zero-frequency MTF value. Thus, while it remains important that $L_y$ be chosen much larger than the y direction resolving distance, open-field normalization will prevent any residual truncation of tails from affecting MTF measurements at nonzero frequencies.

It is common practice to extrapolate the tails of a measured LSF below 1% of its peak value, as this nicely prevents ringing caused by truncation. However, the integrated area under LSF tails determines the low-frequency drop of the MTF. Thus, extrapolation of these tails without knowing how far they actually extend (due to the truncation of the ROI), will result in a LFD that is partly determined by the investigator’s choice of tails. We believe that while extrapolation may be used to reduce ringing, it should only be used in conjunction with open-field normalization to avoid an unspecified normalization error at all frequencies.

The choice of window function affects the extent of spectral leakage and the measured zero-frequency value. Window functions are chosen to reduce spectral leakage by reducing the oscillating tails of the sinc function and, thus, the effect of the sinc function on the Fourier spectrum of the LSF. However, since the window function has values less than unity for nonzero values of the LSF, choice of window will affect the zero-frequency MTF value (central-ordinate theorem) and MTF inflation, if zero-frequency normalization is used. Window shape does not affect MTF values at frequencies above the influence of an LFD when open-field normalization is used. A Hann function was used on both simulated and clinical data in this article, as it satisfies the necessary requirements of a good window function and is commonly used.

Figure 1 shows oscillations in the MTF resultant from spectral leakage caused by the finite ROI. No such oscillations are present in the simulated and clinical results in Figs. 2 and 3. This is due to the frequencies at which the fast Fourier transform algorithm evaluates the Fourier transform. Sampled frequencies depend on the spatial sampling rate and the number of samples in the ROI. Because the ROI size will typically be an integer multiple of the sample spacing, points

<table>
<thead>
<tr>
<th>ROI, $L_x$ (cm)</th>
<th>$A = \text{MTF}^{\text{zf}}(0)$</th>
<th>MTF Inflation $(1-A)/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.97</td>
<td>3%</td>
</tr>
<tr>
<td>10</td>
<td>0.95</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>11%</td>
</tr>
<tr>
<td>1</td>
<td>0.88</td>
<td>14%</td>
</tr>
</tbody>
</table>
in the frequency domain will typically correspond to the zero values of the sinc function in Eq. (9). The oscillations could likely be observed using a non-FFT method to evaluate the Fourier integral.

Most systems exhibit temporal effects such as lag or ghosting to some degree, and it is important to minimize these effects on acquired images. Open-field images should be acquired prior to edge images to prevent residual edge impressions on the open-field images.

The importance of using open-field normalization depends on the extent of the tails in the LSF, or equivalently on whether there is a low-frequency drop in the MTF. Thus, it cannot be specified what the impact will be on any specific system without prior knowledge of the MTF. However, if the clinical system used in this study is typical, zero-frequency normalization may introduce an error of \( \sim 5\% \) at all nonzero frequencies evaluated even when a 10 cm ROI is used. Since the DQE is proportional to the square of the MTF, any error in the MTF would correspond to an error of almost twice that in the DQE.

VI. CONCLUSIONS

Based on a theoretical derivation using linear-systems theory, and validation using both simulated data and measurements on a clinical CsI-based flat-panel detector, we make the following conclusions:

1. The maximum ROI size for all edge-based methods is restricted by nonuniformities of the incident open-field x-ray profile. The open-field normalization technique requires that \( L_x < \frac{\bar{q}(0)}{\max(\bar{q}'(x))}_{|x|<L_x/2} \). No beam nonuniformity effect was observed in measurements made with \( L_x = 1 \) to 15 cm.

2. The measured zero-frequency MTF value is described by Eq. (10) and is less than unity when LSF tails are truncated by the ROI. A value of 0.95 was obtained on a clinical flat-panel system using a 10 cm ROI and a Hann window function.

3. Open-field normalization (without zero-frequency normalization) resulted in validated MTF values of the clinical system at frequencies beyond the low-frequency drop for ROI sizes between 1 and 15 cm.

4. In the limit of a large ROI, open-field and zero-frequency normalization techniques are equivalent.

5. Use of zero-frequency normalization resulted in inflated MTF values by 5% at frequencies beyond the low-frequency drop when using a 10 cm ROI on a clinical system. MTF values were inflated by 11% and 14% when using a 2 cm and a 1 cm ROI, respectively.

It is concluded that a good estimate of the true system MTF, including the low-frequency drop if present, can be achieved using open-field normalization and disregarding the zero-frequency value with the understanding that it would equal unity if LSF tails were not truncated. Open-field normalization does not correct for oscillations caused by truncation to an ROI, but an appropriate window function can be used to reduce these oscillations at the expense of a slight blurring of the MTF.

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APPENDIX A: NONUNIFORM X-RAY DISTRIBUTION

In the presence of a nonuniform incident x-ray distribution, the LSF determined using the edge method, \( lsf^{edg}(x) \), is given by Eq. (5), and includes a small error term in square brackets that has not been previously described. This term is negligible under the condition:

\[
\frac{\bar{q}'(x)}{\bar{q}(0)} < lsf(x).
\]

(A1)

The two components on the left-hand side are very similar with values of approximately \( \bar{q}'(x) / \bar{q}(0) \) or less, and, therefore, the error term is negligible when:

\[
\frac{\bar{q}'(x)}{\bar{q}(0)} < lsf(x).
\]

(A2)

For \(|x|\) less than the LSF width (of order the full width at half-maximum), this condition becomes \( \bar{q}'(x) / \bar{q}(0) < lsf(x)|_{|x|<LSF\text{ width}} \). This is easily satisfied in practice and is not considered to be an important condition. For \(|x|\) much greater than the LSF width, both the LSF and the error term may be very small and it may not be necessary to satisfy Eq. (A1) for all \( x \). However, it is necessary to also ensure the integrated error is small, giving:

\[
\int_{-L_x/2}^{L_x/2} \left[ \frac{\bar{q}'(x)H(x) * lsf(x)}{\bar{q}(0)} - \frac{[\bar{q}(x)H(x) * lsf(x)]\bar{q}'(x)}{\bar{q}(x)\bar{q}(0)} \right] dx < \int_{-L_x/2}^{L_x/2} \text{lsf}(x)dx.
\]

(A3)

Since the error term is of order \( \bar{q}'(x) / \bar{q}(0) \) or less, the integral will be less than \( \max(\bar{q}'(x))|_{|x|\sim L_x/2} L_x / \bar{q}(0) \), and the right-hand side is of order unity, resulting in:

\[
L_x < \frac{\bar{q}(0)}{\max(\bar{q}'(x))|_{|x|<L_x/2}}.
\]

(A4)

This imposes an upper limit on the ROI size, \( L_x \), that depends on \( \bar{q}'(x) \), and is easily satisfied in practice.
APPENDIX B: NORMALIZED SLIT METHOD

The slit method of determining the MTF involves a narrow slit of width \( a \) in radiopaque material placed in front of the detector. The open-field-normalized slit profile, \( S^O(x) \), is given by:

\[
S^O(x) = \frac{kq}{a} \cdot \text{sinc}(x/a) \ast \text{lsf}(x) / kq \ast \text{lsf}(x) = \Pi \left( \frac{x}{a} \right) \ast \text{lsf}(x),
\]

where we have assumed that the input x-ray density is approximately uniform with value \( \bar{q} \) over the slit.

Following similar steps as were used to develop the transfer function for the edge method, for the slit method we obtain:

\[
T^O_{slit}(u) = \left[ a \ast \text{sinc}(au) \ast T(u) \right] \ast L_x \ast \text{sinc}(L_xu) \ast W(u) / a \ast \text{sinc}(au),
\]

where division by \( a \ast \text{sinc}(au) \) is to compensate for the slit width. The slit-method MTF, \( \text{MTF}_{slit}(u) \), is therefore given by:

\[
\text{MTF}_{slit}(u) = \left| \frac{\mathcal{F} \left\{ S^O(x) \Pi \left( \frac{x}{L_x} \right) \ast \text{w}(x) \right\} }{a \ast \text{sinc}(au)} \right| .
\]

To get a properly normalized MTF, however, requires accurate knowledge of \( a \).

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