A moving slanted-edge method to measure the temporal modulation transfer function of fluoroscopic systems

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(Received 14 August 2007; revised 6 April 2008; accepted for publication 7 April 2008; published 23 May 2008)

Lag in fluoroscopic systems introduces a frame-averaging effect that reduces measurements of image noise and incorrectly inflates measurements of the detective quantum efficiency (DQE). A correction can be implemented based on measurements of the temporal modulation transfer function (MTF). We introduce a method of measuring the temporal MTF under fluoroscopic conditions using a moving slanted edge, a generalization of the slanted-edge method used to measure the (spatial) MTF, providing the temporal MTF of the entire imaging system. The method uses a single x-ray exposure, constant edge velocity, and assumes spatial and temporal blurring are separable. The method was validated on a laboratory x-ray image intensifier (XRII) system by comparison with direct measurements of the XRII optical response, showing excellent agreement over the entire frequency range tested (±100 Hz). With proper access to linearized data and continuous fluoroscopy, this method can be implemented in a clinical setting on both XRII and flat-panel detectors. It is shown that the temporal MTF of the CsI-based validation system is a function of exposure rate. The rising-edge response showed more lag than the falling edge, and the temporal MTF decreased with decreasing exposure rate. It is suggested that a small-signal approach, in which the range of exposure rates is restricted to a linear range by using a semitransparent moving edge, would be appropriate for measuring the DQE of these systems. © 2008 American Association of Physicists in Medicine. [DOI: 10.1118/1.2919724]

Key words: modulation transfer function, line spread function, edge spread function, detective quantum efficiency, temporal MTF, x-ray imaging, fluoroscopy, lag, edge

I. INTRODUCTION

Fluoroscopic systems generate a sequence of x-ray images that are displayed for “real-time” visualization of moving patient structures. Risks from radiation exposure are always a consideration in these procedures as average skin dose rates can be 20–50 mGy/min (more for heavy patients) and fluoroscopy times of 60–90 min are not uncommon for some interventional procedures. Optimizing the benefit to risk ratio (and minimizing the risk of skin erythema) requires that the fluoroscopic system produce the best possible image quality for a given patient dose.

The detective quantum efficiency (DQE), based on the noise-equivalent number of quanta, is a metric widely used in radiography for quantifying the “dose efficiency” or signal-to-noise ratio (SNR) performance of x-ray detectors and can be expressed as:

$$\text{DQE}(u) = \frac{\bar{q} G^2 \text{MTF}^2(u)}{\text{NPS}(u)},$$

where DQE(u) is the (one-dimensional) DQE as a function of spatial frequency u, \(\bar{q}\) is the average number of incident x-ray quanta per unit area, \(\bar{G}\) is the average system gain relating \(\bar{q}\) to average pixel value, MTF(u) is the modulation transfer function (MTF) describing (spatial) resolution, and NPS(u) is the Wiener noise power spectrum (NPS) describing (spatial) noise.

Fluoroscopic systems differ from radiographic systems as multiple images are acquired in rapid succession and lag effects (e.g., from phosphors or postprocessing) introduce statistical correlations between images (a form of frame averaging). Siewerdsen et al. showed that, as a consequence of these correlations, it is necessary to use a multidimensional description of noise that includes the temporal domain. Temporal correlations reduce measurements of image noise and, if the temporal domain is not properly interpreted, incorrectly inflate measurements of the DQE.

A number of investigators have used a Fourier approach to describe the temporal characteristics of an imaging system. Levi described the characteristic function of a P-20 phosphor, often used as the output phosphor in x-ray image intensifiers (XRII), as a spatio-temporal quantity and showed that the spatial and temporal characteristics were separable.
using optical methods. Rowlands\textsuperscript{13} measured the temporal MTF of an image intensifier using a spinning disk. Wilson and colleagues\textsuperscript{18–21} showed that Fourier models are useful for describing human spatio-temporal perception when viewing fluoroscopic images. Akbarpour \textit{et al.}\textsuperscript{22} described lag in a phosphor as the (random) delayed release of optical quanta according to a probability distribution function (PDF) and derived theoretical expressions of the corresponding spatio-temporal transfer functions describing noise propagation in the spatio-temporal-frequency domains.

Using the assumption of separability similar to Levi,\textsuperscript{12} it was shown previously that temporal correlations from an XRII reduce the NPS at all spatial frequencies by the factor $\beta$ where:\textsuperscript{9}

$$\beta = a_\nu \int_{-\infty}^{\infty} |T(\nu)|^2 \text{sinc}^2(\pi a_\nu \nu) d\nu = a_\nu \int_{-\infty}^{\infty} \text{MTF}_2^2(\nu) d\nu = \frac{a_\nu}{a_L},$$

and where $T(\nu)$ is the temporal transfer function describing delayed release of light (equal to the Fourier transform of the light emission PDF) expressed as a function of temporal frequency $\nu$, $a_\nu$ is the true image frame-integration time, $\text{MTF}_2^2(\nu)$ is the temporal MTF, and $a_L$, defined by this equation, is the effective temporal aperture of the system and is similar to the information bandwidth integral described by Wagner and colleagues\textsuperscript{23,24} with $|T(\nu)|^2$ acting as an apodization function. Akbarpour \textit{et al.}\textsuperscript{22} showed that temporal correlations affect correlated and uncorrelated components of the NPS differently. Correlations resulting from random delays in the emission of individual secondary quanta, such as delays due to phosphor lag, reduce only the correlated component of image noise. The uncorrelated component (such as noise due to a secondary quantum sink caused by inadequate collection of light photons), usually the smaller component, is not affected. Correlations caused by deterministic processes, such as image frame integration or postprocessing, affect both components. This means that Eq. (2) may be valid only when the uncorrelated noise component is small.

Any measure of the SNR performance of fluoroscopic systems must take these considerations into account. For example, a spatio-temporal DQE could describe the SNR performance (and “dose efficiency”) as a function of both spatial and temporal frequencies, although the physical interpretation and importance for describing observer performance in detection tasks is yet to be established. A simpler approach is to account for the effect of lag in the spatial DQE using $\beta$ from Eq. (2) to scale the NPS,\textsuperscript{9} requiring the system temporal MTF. While the temporal MTF of individual components, such as an image intensifier, may be measured in a laboratory,\textsuperscript{13,25,26} this may not be possible for flat-panel-based systems.

Conceptual use of a temporal MTF remains problematic with many fluoroscopic systems. For example, temporal properties of some systems are exposure-rate dependent,\textsuperscript{27,28} which violates the necessary assumption of linearity for Fourier-based metrics. This may require use of a small-signal approach, limited to an exposure range exhibiting small-signal linearity, as used to accommodate the nonlinear response of film-screen systems.\textsuperscript{3} These issues are addressed further in the Sec. V.

In the past, two methods have been used to determine temporal correction factors to account for the effect of lag on noise: transient-response methods\textsuperscript{8,11,27,29–31} based on image data to monitor the system response to a turn-off or turn-on transient, and integration of the temporal Wiener NPS.\textsuperscript{9,30,32} Transient-response methods are based on measurements at sampling rates determined by the video frame rate (e.g., $\sim 30$ frames/s). This would lead to an undersampled edge profile, and therefore temporal aliasing artifacts, if the temporal response function contains substantial frequency components above one half the frame rate. An example of this type of error would be if the measured system response were dependent on the timing of the turn-off or turn-on transient relative to frame-integration timing. Differentiation to produce the temporal response function could result in further errors.\textsuperscript{33} Integration of the temporal Wiener NPS to determine $\beta$ is valid only when the temporal NPS is proportional to the squared MTF. While this assumption is satisfied for deterministic systems, it is not valid for systems having additional noise sources such as electronic additive noise (generally occurring at low exposure rates). Thus, the NPS method is inappropriate to use if the purpose of a measurement is to identify suboptimal systems or system performance, particularly at low exposure rates.

We are working toward a comprehensive spatio-temporal description of signal and noise, and thus require a method of determining the presampling temporal MTF. In addition to providing an estimate of $\beta$, the presampling temporal MTF is required for more comprehensive metrics, such as a spatio-temporal DQE. Neither the transient-response method, nor the NPS method, can be used for this purpose.

The goal of this article is to describe the theoretical basis and experimental technique of a “moving slanted-edge” method for measuring the presampling temporal MTF of a fluoroscopic system. The method requires access to linearized image data where average pixel value is proportional to incident exposure. It also assumes continuous x-ray exposure and separable spatial and temporal blurring mechanisms. The method is validated using a bench-top laboratory system, but could be implemented in a clinical setting if the above conditions are satisfied.

II. THEORY

The theoretical basis of the moving slanted-edge method to measure the temporal MTF is described using a linear-systems model in which both spatial blur and temporal lag are represented using a spatio-temporal point spread function (PSF), similar to Levi.\textsuperscript{12} We also assume a continuous x-ray exposure.

II.A. Spatio-temporal signal

We represent the system in terms of its three-dimensional (3D) spatio-temporal response\textsuperscript{15,21} with $d(x,y,t)$ describing a presampling output function in space and time:
\[ d(x,y,t) = k \delta(x,y,t) \ast_{x,y} p_{x,y}(x,y,t), \]  

where \( \ast \) is the convolution operator on the dimensions indicated by the subscript, \( k \) is a system gain factor, \( \delta(x,y,t) \) is the incident spatio-temporal distribution of x-ray quanta (quanta mm\(^{-2}\) s\(^{-1}\)), and \( p_{x,y}(x,y,t) \) is the 3D spatio-temporal system PSF. The pixel values, \( d_y \), are samples of \( d(x,y,t) \) evaluated at the center positions associated with each detector element and at the time point corresponding to the end of integration for each video frame.

We assume the PSF is separable in space and time (see Sec. V). Therefore:

\[ p(x,y,t) = p_{x,y}(x,y) \delta(t) \ast_{x,y} p(t) \delta(x,y), \]

where \( p_{x,y}(x,y) \) is the spatial PSF, \( p(t) \) is the temporal PSF, and \( \delta \) is the Dirac delta function. All PSFs are defined to have unity area and include detector aperture effects in \( x, y \) and \( t \), but not aliasing. A simplified graphical representation is illustrated in Fig. 1.

II.B. Moving slanted-edge method

We describe the theoretical basis for the moving slanted-edge method\(^{34} \) as an extension of the slanted-edge method\(^{28-31} \) used to calculate the presampling spatial MTF. Similar to Siewersden and Jaffray,\(^{27} \) a sheet of radiopaque material with a precision semi-infinite edge and moving with constant velocity \( v \) (projected onto the image plane) is placed across a large detector and tilted at a small angle \( \theta \) with respect to the \( y \) axis, creating a moving slanted edge (Fig. 2). We model the relative intensity of the x-ray beam incident on the detector using the 3D Heaviside function, \( H(x,y,t) \), such that a value 0 indicates presence of the edge material, and define the edge to be located at \( (x,y) = (0,0) \) at time \( t=0 \), giving:

\[ H_{\theta}(x,y,t) = \begin{cases} 0 & x - y \tan \theta - vt > 0 \\ 1 & \text{otherwise}. \end{cases} \]  

If \( \bar{\delta}_o \) is the average (uniform) incident fluence rate, the system input can be expressed as \( \bar{\delta}_o H(x,y,t) \) and the corresponding spatio-temporal system output after normalization by an open-field image, \( d_E(x,y,t) \), is expressed as:

\[ d_E(x,y,t) = \frac{k \bar{\delta}_o H(x,y,t) \ast_{x,y} p(x,y,t)}{k \bar{\delta}_o \ast_{x,y} p(t) \delta(x,y)}. \]

Similar to the “synthesized slit” method used to calculate the MTF in the \( x \) direction,\(^{7,36} \) the image signal is integrated over a distance \( Y \) in the \( y \) direction, thus removing the \( y \) dependence, where \( Y \) must be much greater than the resolving distance in the \( y \) direction. Defining \( R_{\theta} \) as the rotation operator by \( \theta \) required to align the edge with the \( y \) axis, and using the convolution property \( \int a \ast b = \int a \int b \),\(^{38} \) for small \( \theta \) we obtain:

\[ d^{\theta}_E(x,t) = \frac{1}{Y} \int_Y R_{\theta} d_E(x,y,t) \, dy = \frac{1}{Y} \int_Y H(x,y,t) \ast_{x,y} R_{\theta} p_{x,y}(x,y) \, dy \times \delta(t) \ast_{x,y} p(t) \delta(x,y) \]

\[ = H(x,t) \ast_{x,t} \frac{1}{Y} \int_Y R_{\theta} p_{x,y}(x,y) \, dy \delta(t) \ast_{x,t} p(t) \delta(x) \]

\[ = H(x,t) \ast_{x,t} \text{lsf}_s(x) \delta(t) \ast_{x,t} p(t) \delta(x), \]

where \( \text{lsf}_s(x) \) is the system line-spread function (LSF) in the \( x \) direction, \( H(x,y,t) \) describes the transmission associated with an edge exactly parallel to the \( y \) axis:

\[ H(x,y,t) = \begin{cases} 1 & x - yt > 0 \forall y \\ 0 & \text{otherwise}, \end{cases} \]

and \( H(x,t) \) is the same function without the \( y \) dependence.

Equation (7) is valid for systems in which the integral through \( p_{x,y}(x,y) \) parallel to the edge is approximately equal to the integral along the \( y \) axis, satisfied for \( \theta \) no more than a
few degrees. Figure 1 illustrates $I_{sf}(x)\delta(t)$ and $p_{v}(t)\delta(x)$ curves. It is shown in Appendix A that Eq. (7) further simplifies to:

$$d_{E}(x,t) = H(x,t) * I_{sf}(x) * p_{v}(t).$$  \hspace{1cm} (9)

The function $d_{E}(x,t)$ describes y line integrals of image edge profiles for all possible $t$ values from a single video frame and is illustrated schematically in Fig. 3(a). For example, the vertical line at $t=0$ in Fig. 3(a) corresponds to the (y-integrated) edge profile in the x direction obtained from a video frame at $t=0$.

II.B.1. Introduction of $\xi$ space

It is now convenient to map all image points to “$\xi$ space,” as illustrated in Fig. 3(b), in which $x_{\xi}$ and $t_{\xi}$ are determined for each pixel, representing estimated distance and time from passing of the edge, respectively. For constant velocity, all points along a line parallel to the edge are mapped to the same point in $\xi$ space with coordinates

$$x_{\xi} = x - \Delta x - vt, \quad \text{and}$$  \hspace{1cm} (10)

$$t_{\xi} = \frac{x - \Delta x}{v} - t,$$  \hspace{1cm} (11)

where $\Delta x$ describes a possible error in $x$ due to not knowing the true edge position. All points in $d_{E}(x,t)$ from a single frame are mapped to the line $d_{E}(x_{\xi} + \Delta x,t_{\xi})$ in $\xi$ space having slope $x_{\xi}/t_{\xi}$ equal to the edge velocity $v$. For convenience, we choose an image with frame integration time ending at time $t=0$ and Eqs. (10) and (11) become

$$x_{\xi} = x - \Delta x, \quad \text{and}$$  \hspace{1cm} (12)

$$t_{\xi} = \frac{x - \Delta x}{v}.$$  \hspace{1cm} (13)

II.B.2. Calculation of temporal MTF

From Eqs. (12) and (13), we have:

$$H(x,t) \rightarrow H(x_{\xi} + \Delta x)$$  \hspace{1cm} (14)

$$I_{sf}(x) \rightarrow I_{sf}(x_{\xi} + \Delta x)$$  \hspace{1cm} (15)

$$p_{v}(t) \rightarrow \frac{1}{v} p_{v}(t_{\xi})|_{t_{\xi}=x_{\xi}/v},$$  \hspace{1cm} (16)

which we use to map Eq. (9) onto $\xi$ space and project onto the $x_{\xi}$ axis, giving:

$$d_{E}(x_{\xi} + \Delta x)|_{v} \rightarrow H(x_{\xi} + \Delta x)|_{\xi} * I_{sf}(x_{\xi} + \Delta x)|_{\xi} * \frac{1}{v} p_{v}(t_{\xi})|_{t_{\xi}=x_{\xi}/v},$$  \hspace{1cm} (17)

the moving-edge profile illustrated in Fig. 3(c). Taking the derivative with respect to $x_{\xi}$ gives:

$$d_{E}(x_{\xi} + \Delta x)|_{v} \rightarrow H'(x_{\xi} + \Delta x)|_{\xi} * I_{sf}(x_{\xi} + \Delta x)|_{\xi} * \frac{1}{v} p_{v}(t_{\xi})|_{t_{\xi}=x_{\xi}/v},$$  \hspace{1cm} (18)

where $H'(x_{\xi} + \Delta x) = \delta(x_{\xi} + \Delta x)$. In the limit $v \rightarrow 0$, this reduces to the (stationary) slanted-edge method giving $MTF_{\xi}(u)$ (Appendix B).

For a nonzero velocity $u_{\xi}$, the effects of spatial and temporal blur are separated by expressing Eq. (18) in the Fourier domain.
\[ T_e(u) \big|_{v=v_o} = T_e(u)T_v(v) \big|_{v=v_o} e^{i2\pi \Delta x u}, \]  
(19)

where \( T_e(u) \big|_{v=v_o} \) is the Fourier transform of \( d'_e(x_\xi + \Delta x) \big|_{v=v_o} \), \( T_e(u) \) is the Fourier transform of \( \text{lsf}_e(x_\xi + \Delta x) \), \( T_v(v) \big|_{v=v_o} \) is the Fourier transform of \( 1/\nu_o p(l) \big|_{x=x_v/v_o} \), and, from Eq. (12), \( u \) represents the spatial frequency corresponding to both \( x \) and \( x_\xi \). The phase \( e^{i2\pi \Delta x u} \) is a result of the Fourier shift theorem. \( ^{38} \)

The temporal MTF is equal to the magnitude of \( T_v(v) \big|_{v=v_o,u} \), given by:

\[
\text{MTF}_T(v) = \left| T_v(v) \big|_{v=v_o,u} \right| = \left| \frac{T_e(u) \big|_{v=v_o}}{T_v(v) \big|_{v=v_o,u}} e^{i2\pi \Delta x u} \right| \]
(20)

where we have made use of the preservation of division property of the modulus operator. \( ^{39} \) \( \left| e^{i2\pi \Delta x u} \right| = 1 \), and \( \left| T_e(u) \big|_{v=v_o} \right| = \text{MTF}_e(u) \) is the system spatial MTF (Appendix B).

### II.B.3. Truncation effects

The above results were derived with an assumption of shift invariance and care must be taken to avoid edge effects near the periphery of the detector. Truncation of the PSF from use of too small a region of interest (ROI) to determine the edge profile will introduce ringing artifacts, \( ^{40} \) although this is not normally seen in practice. Truncation of wide tails in space (or time) will also decrease the PSF area below unity. It can be shown that use of the open-field image to normalize the edge image, as described here, results in a decreased zero-frequency MTF value (central ordinate theorem \( ^{38} \)) that correctly reflects this drop while maintaining correct MTF values at other frequencies. In the present work, it was found that \( T_e(v) \) can have a value below unity at \( v \) = 0 for this reason, but the temporal MTF given by Eq. (20) is correctly normalized and should not be renormalized to unity at \( v = 0 \).

### II.B.4. Practical limitations

Minimum edge velocity is restricted by Eq. (20) as care must be taken to avoid dividing by small values of \( T_e(u) \). Choosing a reasonable minimum value \( T_e(u) = 0.05 \), corresponding to \( u = u_{5\%} \), and a required maximum frequency of importance in \( \text{MTF}_e(v) \), \( v_{\max} \) gives the minimum acceptable edge velocity as:

\[ v_{\min} = \frac{v_{\max}}{u_{5\%}}. \]  
(21)

Using velocities less than \( v_{\min} \) may result in noise in \( \text{MTF}_e(v) \) incorrectly inflating \( \beta \) through Eq. (2).

The maximum edge velocity is restricted by our need to measure data across \( t \). When calculating \( T_e(v) \), the Fourier transform of \( p(t) \), it is assumed that \( p(t) \) is known for all \( t \). In practice, we have measurements over a finite ROI, \( X \), only. We make the reasonable assumption that values of \( p(t) \) must be measured until \( 5\% \) of the peak value is reached, occurring at \( t_{5\%} \) from passage of the edge. Because our ability to map time is limited by the velocity of the edge as it moves through the ROI, we use Eq. (13) to estimate the maximum acceptable edge velocity as:

\[ v_{\max} = \frac{X}{t_{5\%}}. \]  
(22)

At greater velocities, tails of \( p(t) \) may be truncated resulting in underestimated values of \( \beta \).

An incorrectly determined edge position, indicated in Eq. (10) by \( \Delta x \), results in a phase term in Eq. (19). However, this term disappears due to the modulus operator in Eq. (20), indicating that the edge position in each row of the moving-edge image need only be known within a constant offset.

An error in the measurement of \( v_o \) will result in an error in \( t_e \), calculated in Eq. (11). This will result in an error in the calculation of \( \nu \) due to the relationship \( v = u \nu_o \). Because \( T_e(u) \big|_{v=v_o} \) and \( T_e(u) \) do not rely on \( v \), and therefore \( v_o \) as well, an error in knowing the true velocity will affect the calculated \( \text{MTF}_T(v) \) by a scaling along the \( v \) axis.

### II.C. Validation

The moving slanted-edge method of measuring the temporal MTF was validated on an image-intensifier-based fluoroscopic system to show that it gives the same result as a direct measurement of the XRII optical output response function. Lag in these systems is predominately due to phosphor characteristics and frame integration time. Direct measurement of the XRII optical response \( ^{27,41–43} \) was obtained from measurements of the light output as \( x \) rays were turned off quickly. We represent a semi-infinite \( x \)-ray exposure incident on the detector and ending at \( t = 0 \) as \( q_\infty(t) = q_\lambda(t) \) where \( \lambda(t) \) is the relative turn-off profile. The normalized output signal from a photodiode at the XRII output, \( s(t) \), is given by:

\[
s(t) = \frac{k_1 q_\infty(t) \ast p(t)}{k_1 q_\lambda(t) \ast p(t)} = \lambda(t) \ast p(t), \]

(23)

where \( p(t) \) is the temporal PSF of the XRII’s phosphor and \( k_1 \) is a constant of proportionality. Ideally, the turn-off profile \( \lambda(t) \) would be a step function, but is measured independently using a second photodiode optically coupled to a Gd\(_2\)O\(_2\)S:Tb phosphor (having a faster response than the XRII). This monitor signal, \( m(t) \), is expressed as:

\[
m(t) = \frac{k_2 q_\lambda(t) \ast p_{\text{Gd}}(t)}{k_2 q_\lambda(t) \ast p_{\text{Gd}}(t)} = \lambda(t) \ast p_{\text{Gd}}(t), \]

(24)

where \( p_{\text{Gd}}(t) \) is the temporal PSF of Gd\(_2\)O\(_2\)S:Tb. It is convenient to express the derivatives with respect to time, \( s'(t) = \lambda'(t) \ast p(t) \) and \( m'(t) = \lambda'(t) \ast p_{\text{Gd}}(t) \), and to move into the Fourier domain, giving:

\[
S(v) = \Lambda(v) T_e(v) \quad \text{and} \quad \]

(25)
\[ M(v) = \lambda_e(t)T_{Gd}(v), \]  
where \( S(v), T_{Gd}(v), \lambda_e(t), \) and \( T_{Gd}(v) \) are the Fourier transforms of \( s'(t), p(t), \lambda'_e(t), \) and \( p_{Gd}(t) \) respectively. Therefore:

\[ T_{Gd}(v) = S(v) \frac{T_{Gd}(v)}{M(v)}. \]  

Scaling by \( T_{Gd}(v)/M(v) \) deconvolves out nonstep-like characteristics of the turn-off profile \( \lambda_e(t) \) (within the limitations of linear-systems theory). The dominant luminescent decay of Gd_2O_2S:Tb is exponential with a time constant \( \tau = 0.558 \text{ ms}, \) giving \( TGd \) \( = 1/\sqrt{1+(2\pi\nu)^2} \). The direct estimate of the system temporal MTF, accounting for frame integration time \( a_t \), is therefore given by:

\[ \text{MTF}_d(v) = |T_{Gd}(v)||\text{sinc}(\pi a_tv)|. \]  

It is important to note again that by normalizing the measured signal to an open-field response, it is not appropriate to normalize \( \text{MTF}_d(v) \) by the \( v=0 \) value.

III. METHODS

All results were obtained using a continuous fluoroscopic exposure and a laboratory fluoroscopic system consisting of a four-year-old Dunlee intensifier (model TH715-3N:91 01 2693), Prosilica charge-coupled device (CCD) camera (model EC1350) operated in frame-transfer mode \(^{35} \) (29.97 frames per second, frame-integration time of \( a_t = 32.0 \text{ ms} \)), pixel size of 0.28 mm \( \times \) 0.28 mm in the image plane, and a 1.3-mm-thick lead plate with precision-machined edge. The edge was supported in a custom-made assembly placed against the detector input and moved under servo-motor control. The x-ray tube was operated at 50 kV and the beam hardened using 0.8 mm of Cu. All images were dark-field subtracted, open-field normalized, corrected for geometric distortions, \(^{36} \) and a Hann window was applied to all data before calculating the fast Fourier transform (FFT). The frame with the edge closest to the detector center was used to calculate the temporal MTF. Measurements obtained by covering the detector \( (v_o > 0) \) are referred to as “falling edge,” and similarly those obtained by uncovering the detector \( (v_o < 0) \) are “rising edge.”

III.A. Moving slanted-edge method

The moving slanted-edge method can be implemented with the following steps:

1. Calibrate pixel size in image plane. If necessary, geometric distortions should be corrected to ensure uniform pixel size throughout the image.
2. Measure the presampling (spatial) MTF using a stationary slanted edge. This is equivalent to the (stationary) slanted-edge method.
3. Capture a frame of the slanted edge moving with constant velocity \( v_o \) chosen such that \( v_{\text{min}} \leq v_o \leq v_{\text{max}} \) using Eqs. \((21)\) and \((22)\), and normalize to an open-field image.
4. Determine the edge positions (within a constant offset) in each row. This can be achieved by finding the location of fastest signal change and fitting a straight line to results from all rows. Average position change between two images can be used to determine \( v_o \).
5. For each pixel in the frame, calculate \( x_\xi \), an estimate of the distance from the edge given by Eq. \((12)\), and rebin the data \(^{35,36} \) to produce the oversampled (moving) edge profile, \( d'_\xi(x_\xi+\Delta x)|_{x=x_\xi} \), given by Eq. \((17)\).
6. Differentiate the edge profile with respect to \( x_\xi \) to produce \( d''_\xi(x_\xi+\Delta x)|_{x=x_\xi} \) in Eq. \((18)\).
7. Take the modulus of the Fourier transform of \( d''_\xi(x_\xi+\Delta x)|_{x=x_\xi} \) to produce \( |T_{\xi}(v)|_{\text{max},\mu} \) in Eq. \((19)\).
8. Calculate the presampling temporal MTF, \( \text{MTF}_d(v) \), by dividing \( |T_{\xi}(v)|_{\text{max},\mu} \) by the (spatial) MTF, \( |T_{\xi}(v)|_{v=0,\text{max},\mu} \) in Eq. \((20)\).

III.B. Validation

The spatial MTF was determined using an average of 100 edge-image frames. The temporal MTF was calculated using both rising and falling-edge profiles with \( v_o = 37 \text{ cm/s} \) at exposure rates of 27 and 250 \mu R/frame. Validation was performed by comparison with the temporal MTF calculated using direct measurements of the XRII optical output using a photodiode detector. A second photodiode was optically coupled to a Gd_2O_2S:Tb scintillator and placed in the x-ray beam to monitor beam intensity and determine \( m(t) \). Both signals were digitized simultaneously at 400 kHz and subsequently rebinned to an effective sampling rate of 2 kHz. Visual inspection of the monitor signal showed that a 2 s exposure was required to achieve steady-state conditions. In all experiments, a 6.3 s exposure was used (the longest available with the generator used) after which the x-ray exposure was turned off abruptly. The signals were differentiated and a five-point average used on the slowly changing tails of the curves to reduce noise. The FFT was calculated and the temporal MTF determined after multiplication by the sinc function using Eq. \((28)\). Edge-velocity magnitudes of 10, 19, 28, 37, 46, 56, and 74 cm/s were used to verify the useful velocity range.

III.C. Nonlinear considerations

III.C.1. Rising versus falling-edge profiles

The temporal MTF was determined using both rising and falling-edge profiles to determine the impact of this difference at 27 \mu R/frame.

III.B.2. Incident exposure rate

Exposure-rate dependence of the temporal MTF was determined for incident exposure rates ranging from 0.3 to 81.2 \mu R/frame, representing low to very high fluoroscopic rates. \(^{37} \) Measurements were made in the stated order to minimize the impact of potential exposure-history effects.
IV. RESULTS

IV.A. Moving slanted-edge method

Figure 4 shows the system spatial and temporal MTF curves obtained using the moving slanted-edge method based on Eq. (20). The spatial MTF has a low-frequency drop of approximately 7%. It is important to note that, since the edge image is normalized by an open image, the MTF curve should not—and indeed must not—be renormalized at zero frequency as this would incorrectly inflate MTF values at all other spatial frequencies. The zero-frequency value is less than unity due to the fact that the LSF tails are truncated by the finite region of interest (≈8 cm) used to calculate the MTF.

The temporal MTF reflects operation of the entire system, including both XRII and CCD camera in our experiment, and a zero-value frequency, determined by frame integration time, is expected at 1/\(a_t\)=31.25 Hz.

IV.B. Validation

Figure 5 shows the normalized monitor signal measuring the x-ray beam profile, \(m(t)\), and the normalized photodiode signal at the XRII optical output, \(s(t)\), as measured in a single falling-edge trial at 27 μR/frame, and associated Fourier functions from these profiles. Also shown is the theoretical transfer function for \(\text{Gd}_2\text{O}_2\text{S}:\text{Tb}^{44}\) and the XRII’s temporal transfer function, as calculated using Eq. (27).

A comparison of the moving slanted-edge method (average of three independent trials) with the direct estimate of the temporal MTF based on \(\text{T}_I(v)\) (average of six independent trials) is shown in Fig. 6. Agreement is seen within measurement precision for both rising and falling-edge measurements, and exposure rates of 27 and 250 μR/frame, at all temporal frequency values. While error bars indicate the standard deviation of each frequency value on all curves, they are too small to be seen easily on the moving slanted-edge results due to superior noise characteristics of this method.

Figure 6 shows that while the temporal characteristics are highly dependent on exposure conditions, the moving slanted edge and direct methods give the same result for the same conditions. It also shows that the function \(\text{T}_I(v)\), describing lag in the XRII phosphor, can drop by as much as 40% at 30 Hz under some conditions. Thus, while the sinc function resulting from frame integration time dominates the system temporal response, phosphor lag can have a substantial effect and cannot be ignored. Attempts to compare the two methods at typical fluoroscopic exposure rates failed due to poor SNR in direct-method measurements of \(m(t)\).

Table I shows values of \(\beta\) calculated from Eq. (2) using data in Fig. 6. The two methods agree within measurement...
precision for both rising and falling-edge profiles at the two exposure rates. The value $\beta$ is determined from the integral of $MTF_t(\nu)$ and, to limit bias due to noise integration, was numerically integrated only until the curve decayed to approximately 0.005, corresponding to $\pm 100$ Hz. Table I shows that image noise power is reduced by 9% to 31% due to lag in the test system with the greatest reduction occurring at the lower exposure rate using the rising-edge profile.

Calculated values and uncertainties of $\beta$ using the moving slanted-edge method are shown in Fig. 7 as a function of edge velocity. The theoretical range of acceptable edge velocities is $7–56$ cm/s ($v_{\text{min}}=100$ Hz, $u_{5\%}=1.4$ cm/s in Eq. (21), $X=8.4$ cm, and $t_{5\%}=0.15$ s in Eq. (22) for both rising and falling-edge profiles). Throughout this range, trials at each velocity show deviations in $\beta$ of less than 3% from the mean, and mean values at each velocity deviate from the weighted mean across all velocities by less than 4%. Measurements taken near $v_{\text{min}}$ show an upward trend in $\beta$ and values taken above $v_{\text{max}}$ show a downward trend, consistent with the prediction in Sec. II B 4.

IV.C. Nonlinear considerations

IV.C.1. Rising versus falling-edge profiles

It was observed that rising-edge $\beta$ values were consistently smaller than falling-edge values (ratio $=0.911 \pm 0.009$) for all velocities (Fig. 7), consistent with the observation that rising profiles showed slightly more lag than did falling profiles and responsible for the rising-edge temporal MTF decreasing slightly faster with frequency, as seen in Fig. 8. While the difference is modest, it is responsible for the $\sim 9\%$ difference in $\beta$ values shown in Fig. 7. Both curves

![Graph showing comparison of moving slanted-edge method, MTF_t(\nu) from Eq. (20) (solid circles, v_c=37 cm/s, average of three independent trials), with the direct method, MTF_d(\nu) in Eq. (28) (open circles, average of six independent trials), and temporal MTF of the XRII, $|T_T(\nu)|$ (crosses, average of six trials). (a) 27 $\mu$R/frame, falling-edge profile; (b) 27 $\mu$R/frame, rising-edge profile; (c) 250 $\mu$R/frame, falling-edge profile; and (d) 250 $\mu$R/frame, rising-edge profile. Error bars on each point indicate the standard deviation of the curves at each frequency.]

**Table I.** Comparison of $\beta$ calculated from Eq. (2) using both the moving slanted-edge method (mean and standard deviation of three trials) and the direct method based on the XRII optical decay curve (mean and standard deviation of six trials).

<table>
<thead>
<tr>
<th>Profile</th>
<th>Exposure rate (\mu R/frame)</th>
<th>Direct method</th>
<th>Moving slanted-edge method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rising edge</td>
<td>27</td>
<td>0.69±0.06</td>
<td>0.738±0.003</td>
</tr>
<tr>
<td>Falling edge</td>
<td>27</td>
<td>0.81±0.04</td>
<td>0.8155±0.0006</td>
</tr>
<tr>
<td>Rising edge</td>
<td>250</td>
<td>0.865±0.013</td>
<td>0.8726±0.0011</td>
</tr>
<tr>
<td>Falling edge</td>
<td>250</td>
<td>0.912±0.009</td>
<td>0.906±0.003</td>
</tr>
</tbody>
</table>

Medical Physics, Vol. 35, No. 6, June 2008
are the mean of three independent trials with error bars indicating the standard deviation of each frequency value.

IV.C.2. Incident exposure rate

Both rising and falling-edge temporal MTF curves showed a dependence on incident exposure rate (Fig. 9). Temporal MTF values increase at all frequencies with increasing exposure rate, indicating that lag is greatest at the lower rates. The change is not a simple scaling. The XRII temporal MTF in Fig. 9(b) shows a low-frequency drop of 20% at low rates that disappears at high rates. This suggests a long time-constant component in the response that is seen only at the low rates. Also, the low exposure rate results show an additional decrease of over 35% at frequencies greater than 10 Hz, consistent with the idea that lag results from multiple time constants with different exposure-rate dependencies.

Rising-edge results showed greater lag than falling-edge results. However, as the exposure-rate is increased, agreement between rising and falling-edge measurements improved, as noted by the converging $\beta$ values in Fig. 9(c).

V. DISCUSSION

The use of Fourier methods to describe the temporal response of a detector requires several assumptions that are not necessarily well satisfied. For example, one requirement of shift invariance (in time) is that the response be independent of exposure history, although many detectors have a known
history dependence. This issue must be addressed before temporal MTF concepts can be physically meaningful. In the present work, no history dependence was observed, but neither was it specifically investigated.

The assumption of linearity can also be problematic. For a spatio-temporal system, this requires that \( p_{x,y}(x,y,t) \) in Eq. (3) be independent of exposure rate. The present study shows an exposure-rate dependence of the temporal MTF (consistent with expectation and likely explaining the difference between rising and falling-edge results), and means the system is not linear over the range of exposure rates resulting from the opaque edge. Therefore, these measurements cannot fully describe the temporal response of the system, but only the specific response for the conditions under which they were tested. One possible solution is to adapt small-signal methods to the measurement of the temporal MTF by using a semitransparent edge. Under small-signal conditions, exposure-rate dependencies should become negligible, and rising and falling-edge results are expected to be equivalent. The small-signal temporal MTF, measured under the same exposure-rate conditions used to measure the NPS, is expected to provide the correct \( \beta \) value to use for DQE calculations in Eq. (2), and accurately describes the system over this small range of exposure rates. The present study, using an opaque edge, lays the foundations for a small-signal method using a moving slanted edge (theory and procedure) with validation by direct measurements of the XRII optical response under the same conditions.

The assumption of spatio-temporal separability is vital to our method. The temporal correction factor \( \beta \), which was motivation for this work, was also derived based on an assumption of separability. Separability can be tested by ensuring that the (spatial) MTF remains constant under varying temporal conditions, and our results showed changes in temporal lag but not spatial blur. The assumption of separability is likely valid for most x-ray based systems, as different physical processes are responsible for these two types of blurring, and Levi has confirmed the spatio-temporal separability of P-20 phosphors.

The direct method requires a fast turn-off of the beam. Deconvolution of the turn-off profile is not needed if a turn-off time of \(<1 \text{ ms}\) [based on Fig. 5(a)] can be achieved. One option was to use a mechanical shutter, but this would require a shutter speed of \(~10 \text{ m/s}\). Rather, the generator turn-off profile was used and measured independently with a \( \text{Gd}_2\text{O}_2\text{S}:\text{Tb} \) detector for deconvolution using Eq. (27). A potential concern with this approach is the different spectral response of this detector relative to CsI if there is a change in high voltage due to cable capacitance during the turn-off transition. However, a narrow spectrum, such as the one used (50 kVp with 0.8 mm Cu filtration), will minimize this effect and the fact that the measured turn-off profile closely approximated the expected decay of \( \text{Gd}_2\text{O}_2\text{S}:\text{Tb} \) leads us to believe that the effect was negligible. Deconvolution using the turn-off profile was implemented to remove the remaining effects of minor deviations from an ideal turn-off profile.

Calculations of the XRII temporal response (\( |T_1(\nu)| \)) are highly sensitive to the choice of temporal integration window as division by the small sinc-function values at frequencies near and above 30 Hz can have a large effect. A direct measurement of the XRII temporal response at high frequency is likely more appropriate to determine the XRII temporal MTF.

Calculation of the temporal correction factor \( \beta \) requires the numerical integration of \( \text{MTF}_2^2(\nu) \) within finite limits. Noise increases with frequency due to the differentiation step and hence if these limits are too large, noise will inflate the integral, while if these limits are too low, truncation will underestimate \( \beta \). Limits of \( \pm 100 \text{ Hz} \) were chosen as the value of \( \text{MTF}_2^2(\nu) \), including noise, was \(~0.005\) at this frequency. The truncation effect is known to be less than \(~3\%\) from a numerical integration of \( \text{sinc}^2(\pi\nu) \) and the error in \( \beta \) is even less as the temporal MTF always decreases faster than a sinc function.

The (stationary) slanted-edge method requires that the edge be accurately aligned with the incident beam. This is not possible with a moving edge and results in a changing penumbral effect. Maximum penumbral blur was estimated as \(0.02 \text{ mm} \) (7\% of pixel size), occurring at the edge of the ROI, 4.2 cm from the center of the XRII. At \( v_o = 37 \text{ cm/s} \), this could introduce a 0.07 ms temporal blur, which is considered negligible.

The analysis assumed all pixels in a specified image frame have the same integration start and stop times. This is a reasonable assumption for the CCD camera used as it operated in a frame-transfer mode. However, some detectors may stagger the integration periods, resulting in a curvature or other distortion of the imaged edge, and require an additional term in Eqs. (10) and (11) and derived equations. Interlaced video readout, in particular, may be problematic as spatial blur between image lines will introduce a complicated sharing of image information in both space and time.

The calculated zero-frequency values of the temporal MTF in Fig. 9 are less than unity due to a truncation artifact caused by the finite region of interest (Sec. II B 3), and the smallest value of 0.8 is noted for the exposure rate of 0.3 \( \mu \text{R}/\text{frame} \). Thus, normalization of the temporal MTF by the zero-frequency value will incorrectly inflate values at all other frequencies by 20\% for this exposure rate. This problem was avoided by normalizing the edge image to an open-field image.

Figure 9(c) shows a trend of decreasing lag with increasing exposure rate. This trend is opposite to results observed by other investigators using an amorphous silicon flat-panel imager. While the explanation for this difference is not known, it should be noted that our measurements were made using an XRII-based system. Lag in an XRII is influenced by additional factors not present in a flat-panel imager, such as charge buildup on the output phosphor. The moving slanted-edge results were validated by direct measurements on the XRII output, suggesting that substantial differences may ex-
The moving slanted-edge method provides accurate measurements of the presampling temporal MTF of the entire fluoroscopic system, and unlike the direct method, does not require access to intermediate components. This is critical for some systems, such as flat-panel detectors, where a direct measurement of light from the phosphor is not physically possible. Since only image data is used, the method can be implemented in a clinical setting provided the linear (or linearized) digital images are accessible. This may not be possible on all systems (without assistance from the manufacturer), and all nonlinear and nonstationary postprocessing algorithms will have to be disabled before a Fourier approach is possible. With access to both raw and postprocessing data (using only linear and stationary algorithms), the temporal influences resulting from random delays of quanta versus these deterministic processes can be compared. While this paper has focused on the application of this method to fluoroscopic systems, temporal MTF considerations may also be important in any detector involving rapid data readouts, such as modern computed tomography systems. The moving slanted-edge method can also be adapted for use with these systems, provided all previously outlined assumptions and requirements of the method can be satisfied.

The theoretical development assumed continuous fluoroscopic exposure, a requirement for the direct method used for validation. Extension to pulsed fluoroscopic systems is currently under investigation by generalizing the theoretical development and representing the incident exposure as a cyclostationary random process.

VI. CONCLUSIONS

We make the following conclusions:

(1) The theoretical basis of the moving slanted-edge method has been developed and an experimental method has been summarized (eight steps in Sec. III A). The method gives the same results as those derived from direct measurements of the XR II phosphor optical output.

(2) Equations (21) and (22) provide the acceptable range of edge velocities and only a single velocity is needed for the measurements.

(3) The fluoroscopic system’s temporal response is dependent on exposure rate; lower exposure rates result in more lag.

(4) Estimates of the temporal MTF using rising and falling-edge profiles differ. More lag is present in rising-edge measurements.

The moving slanted-edge method provides a needed tool for the continued understanding of temporal properties of fluoroscopic systems, and the development of a Fourier-based metric of system performance. The method is limited to fluoroscopic systems that have a separable spatio-temporal response, can be operated in continuous fluoroscopic mode, and for which the linearized output is available.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the Canadian Institutes of Health Research. S.N.F. acknowledges a scholarship from The University of Western Ontario. The authors would also like to thank Nicolas Jankovic, Dr. Sergey Lazarev, Sujeewan Velupillai, and Hristo Nikolov for their help with experiments.

APPENDIX A: EXPANSION OF CONVOLUTIONS

The first convolution in Eq. (7) can be expanded as:

\[ H(x,t) \ast_x \text{lsf}_f(x) \delta(t) \]

\[ = \int \int H(x',t') \text{lsf}_f(x - x') \delta(t - t') dx' dt' \]

\[ = \int \int H(x',t') \text{lsf}_f(x - x') dx' \delta(t - t') dt' \]

\[ = \int H(x',t) \text{lsf}_f(x - x') dx' \]

\[ = H(x,t) \ast_x \text{lsf}_f(x). \]  \hspace{1cm} (A1)

Substituting this result back into Eq. (7) and expanding the second convolution gives:

\[ d^p(x,t) = H(x,t) \ast_x \text{lsf}_f(x) \ast_x p(t) \delta(x) \]

\[ = \int \int \int H(x',t') \text{lsf}_f(x' - x'') dx'' p(t - t') \]

\[ \times \delta(x - x') dx' dt' \]

\[ = \int \int \int H(x'',t') \text{lsf}_f(x' - x'') dx'' \]

\[ \times \delta(x - x') dx' p(t - t') dt' \]

\[ = \int \int H(x'',t') \text{lsf}_f(x - x'') dx'' p(t - t') dt' \]

\[ = H(x,t) \ast_x \text{lsf}_f(x) \ast_x p(t). \]  \hspace{1cm} (A2)

APPENDIX B: STATIONARY EDGE LIMIT

In this appendix it is shown that the moving slanted-edge method is equivalent to the (stationary) slanted-edge method used to measure the (spatial) MTF. In the limit of \( v \to 0 \) in Eq. (18), we obtain:

\[ d^s_f(x_f + \Delta x) \bigg|_{v=0} = \lim_{v \to 0} \text{lsf}_f(x_f + \Delta x) \ast_{x_f} \frac{1}{v} \bigg|_{u=x_f/v} \]

\[ = \text{lsf}_f(x_f + \Delta x) \ast_{x_f} \delta(x_f) \]

\[ = \text{lsf}_f(x_f + \Delta x). \]  \hspace{1cm} (B1)

The Fourier transform of \( d^s_f(x_f + \Delta x) \bigg|_{v} \) is \( T_f(u) \bigg|_{v} \). In the limit \( v \to 0 \), the (spatial) MTF in the \( x \) direction is given by:
\[ \text{MTF}_x(u) = |T_x(u)|_{u=0} = |T_x(u)|. \]  

(2)

\[ \text{MTF}_x(u) = |T_x(u)|_{\epsilon=0} = |T_x(u)|. \]  

(2a)

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