Angular-dependent coherent scatter measured with a diagnostic x-ray image intensifier-based imaging system

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Low-angle scatter of x rays at diagnostic energies is primarily coherent. This coherence gives rise to interference effects resulting in x-ray diffraction patterns that are characteristic of the scattering material. A method is described of imaging these low-angle (0°–10°) x-ray diffraction properties of tissue specimens using a diagnostic x-ray beam and image intensifier-based system. The coherent-scat ter cross sections of several materials measured this way are presented. It is shown theoretically that the measurements made with this system can be expressed as the mono-energetic cross section “blurred” by the x-ray spectrum using a linear superposition integral. Experimental results using aluminum powder confirm this. Using a 70 kVp x-ray beam filtered with gadolinium to reduce the spectral width, materials such as water, Lucite, and hydroxyapatite all have significantly different diffraction patterns. The cross sections determined from this analysis form the basis of a unique method of characterizing and identifying tissue samples according to their atomic structure rather than x-ray attenuation properties. © 1996 American Association of Physicists in Medicine.

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I. INTRODUCTION

Since 1912, when von Laue discovered that crystals diffract x rays, diffraction studies have been used by scientists to investigate the structure of matter at the molecular level in a wide range of materials. Crystallography is the most common application, where x-ray diffraction is used to determine the positions of the atoms within a crystal structure.

Coherent scatter is strongly forward peaked at diagnostic energies and dominates over Compton scatter at low scattering angles (0°–10°).1 Due to the coherence between x rays scattered from different sites in the scattering medium, interference results and x-ray diffraction occurs. The terms coherent scatter and x-ray diffraction will be used interchangeably in this article. In general, coherent scatter constitutes approximately 10% of the total interaction cross section, and hence it is frequently ignored in general radiography. However, this strongly forward-peaked nature compensates for its modest cross section, and the intensity of coherent scatter can be dramatically greater than the intensity of Compton scatter at these small angles. This enables the low-angle coherent scatter from an object to be detected without significant contamination from Compton scatter, allowing the determination of the unique information that coherent scatter provides.

The usefulness of coherent scatter is not limited to crystallography. It can provide important information about biological materials as well. For example, coherent scatter was first used to determine bone-mineral density by Puunalainen et al.2 They used 59.5 keV photons and measured the ratio of coherent to Compton scatter at 90° to determine the average charge number of their samples. The coherent to Compton scatter ratio was found to be proportional to the hydroxyapatite content of the samples. More recently, a method of measuring bone-mineral content (BMC) using a measurement of the coherent-scat ter cross section alone has been developed elsewhere and verified by ashing and histomorphometry.3,4 It has been suggested that measurements of coherently scattered x rays can result in a more accurate measurement of BMC than methods such as radiography, photon absorptiometry, and quantitative CT,4,5 as these other methods are either directly or indirectly bone-density dependent. The coherent-scat ter method is based on the fact that different materials generally produce different diffraction patterns. For example, materials such as bone and fat have been shown to have distinctly different cross sections.6 In particular, hydroxyapatite creates a distinctive diffraction pattern due to its polycrystalline nature, which results in a significant difference between the cross sections of healthy and osteoporotic bone,7 in spite of the fact that osteoporotics can have equal or greater bone density as compared to healthy subjects.8–10

Another potential biological area of interest is in scatter measurements from breast tissues. Evans et al.11 used a 60 kVp x-ray beam from a copper anode x-ray tube filtered with 0.5 mm Cu to investigate small-angle scatter from 19 different samples of breast tissue (e.g., adipose tissue, fibroglandular tissue, carcinoma, fibroadenoma, etc.). They found that variations in the peak position (3.43°–6.01°) appeared to
provide greater sensitivity to changes in tissue type than direct transmission measurements.

It is often assumed that mono-energetic x-ray sources are required for high-quality measurements of these diffraction patterns. This article describes a method of measuring coherent-scatter cross sections of tissue specimens using a diagnostic x-ray beam and image intensifier-based system. It is shown how the ideal scatter cross section is "blurred" by the spectrum used, producing a spreading of approximately 14% in the measured cross sections. Experimental scatter patterns from several materials are compared with theoretical expectations or mono-energetic measurements. These results are used for imaging BMC in a tomographic slice using a method described elsewhere\(^\text{12}\) and similar to a method described by Harding et al.\(^\text{13-15}\)

II. THEORY

A. Coherent scatter

Coherent scatter is described classically as the interaction between the electric field associated with the x-ray beam and the electric field associated with the electron charge distribution in the material. The electrons are set oscillating and subsequently emit radiation of the same wavelength as the incident beam. Interference effects between the radiation emitted by the different electrons result in a characteristic diffraction pattern for a given material. This interference occurs between radiation scattered by electrons in the same atom, electrons of different atoms within a molecule, and electrons within different molecules.

If the atoms within a material are rigidly fixed to one another in a crystal lattice, the phase relationships between radiation emitted by electrons within different atoms are constant. As a result, constructive interference between scattered rays occurs only at certain well-defined angles given by the Bragg condition:\(^\text{16}\)

\[
\lambda = 2d \sin \theta_B,
\]

where \(\lambda\) is the x-ray wavelength, \(d\) is the spacing between crystal planes, and \(\theta_B\) is the Bragg angle. The scattering angle \(\theta\) (Fig. 1) between the transmitted primary beam and the scattered ray is twice the Bragg angle. The set of well-defined scattering angles for a particular material allows crystallographers to determine the crystal structure of the material.

Amorphous materials (lacking in well-defined crystalline structure) do not give such well-defined scatter conditions, but they do give different diffraction patterns which are characteristic of the particular material. The differential coherent-scatter cross section for an amorphous material is given by the product of the Thomson cross section \([r_e^2/2](1 + \cos^2 \theta)\) and a squared form factor \([F^2(x, Z)]\) which describes the interference effects between radiation scattered by different electrons within the material:\(^\text{17}\)

\[
\frac{d\sigma_{coh}}{d\Omega}(\theta) = \frac{r_e^2}{2}(1 + \cos^2 \theta)F^2(x, Z),
\]

which is expressed in terms of \(m^2/\)atom/steradian, where \(r_e\) is the classical electron radius \((2.82 \times 10^{-15}\text{m})\), \(x\) is the momentum transfer argument, given by

\[
x = (1/\lambda) \sin(\theta/2),
\]

and the form factor \(F(x, Z)\) is equal to the Fourier transform of the electron charge distribution\(^\text{18}\) and is therefore a property of the material only. If the material is not amorphous (i.e., the charge distribution is not spherically symmetric), the form factor will depend on the azimuthal angle \(\phi\) (Fig. 1) as well as upon \(\theta\). Note that the momentum transferred to the x-ray photon is equal to \((2h/\lambda)\sin(\theta/2)\) and therefore \(x\) is not actually a momentum; rather, \(x\) is arrived at by dividing the momentum transfer by twice Planck’s constant.

B. Theoretical coherent-scatter diffraction patterns

X-ray diffraction patterns result from interference effects of the coherent scatter. Theoretical calculations of these patterns are in general extremely difficult to evaluate. In this section, a method of theoretically describing the diffraction from a powdered polycrystalline material is discussed. The development is somewhat non-standard in that the scatter distribution is described by the cross section per unit angle \(d\sigma/d\theta\) rather than by the cross section per unit solid angle \(d\sigma/d\Omega\), as a function of \(\theta\). It is shown that the cross-section measurement made with a poly-energetic x-ray spectrum can be represented as a linear superposition integral or a "non-stationary convolution" of the mono-energetic cross-section measurement (scaled by a weighting factor) with a function that is directly related to the x-ray spectral shape. The results are used later to compare the theoretical diffraction with experimental measurements using aluminum. Note that the term convolution only strictly applies to the situation where the superposition kernel is stationary, while in this case the kernel is spatially variant.

The main importance of this section is in the evaluation of the poly-energetic cross-section measurement. Much of the development for the mono-energetic cross-section measurement is standard textbook material,\(^\text{19}\) however, it is repeated here for clarity as well as to facilitate the extension to the
poly-energetic case. In addition, it appears in a modified form due to the fact that it is the cross section per unit angle that is evaluated.

1. Mono-energetic beam

The diffraction pattern produced by a powdered polycrystalline material obtained using a thin mono-energetic beam of x rays consists of a series of sharp circles (sometimes referred to as "lines") about the axis of the incident beam. Each circle corresponds to scatter from a particular set of crystal planes. Since the material is powdered, the crystals are randomly oriented and thus the crystal planes appear at all angles relative to the incident beam. It is this random orientation that causes sets of planes with the same plane spacing but different orientations to scatter to different locations, which on average form a circle. The intensity within these circles is independent of the azimuthal angle \( \phi \) (Fig. 1). The total number of x rays scattered into the \( j \)'th circle, \( v_j \), is given by

\[
v_j = N_0 I_j,
\]

where \( I_j \) is the probability of an x ray being scattered by the \( j \)'th set of crystal planes and \( N_0 \) is the number of x rays incident on the object. This scattering occurs at a particular angle \( \theta_j \) which is dependent on the x-ray wavelength \( \lambda_M \) used. The \( M \) subscript indicates the mono-energetic nature of the x-ray beam. Through the Bragg condition [Eq. (1)],

\[
\lambda_M = 2d_j \sin(\theta_j/2),
\]

where \( d_j \) is the plane spacing for the \( j \)'th set of crystal planes. From the form of the coherent-scatter cross section [Eq. (2)], \( I_j \) is expected to depend on \( F_j^2(x_j, Z) \times (1 + \cos^2 \theta_j) \). For an amorphous, isotropic material, the coherent-scatter cross section alone describes the scatter intensity as a function of scattering angle. However, for a crystal, the observed scatter also depends on the crystal orientation. The crystal must be aligned to satisfy the Bragg condition [Eq. (1)] in order to have detectable scatter. If the sample is a polycrystalline powder, the random orientation of the particles will ensure that some particles satisfy this condition. However, the probability \( I_j \) of an x ray being scattered by the \( j \)'th set of crystal planes also depends on three additional factors.

The first of these factors is the multiplicity factor \( p_j \). Within a crystal there are several sets of parallel planes which give scatter at the same angle and are therefore indistinguishable. Since they contribute to the same line in the diffraction pattern, the probability of satisfying the Bragg condition for this particular scattering angle must be multiplied by the multiplicity factor.

The second factor which affects the total number of scattered x rays is due to the finite size of the crystal fragments within the polycrystalline material and the fact that the x rays in the incident beam are diverging and thus do not travel along parallel trajectories. As a result, the diffraction lines are not perfect \( \delta \)-functions, but are actually narrow peaks. For example, a hypothetical line shape is shown in Fig. 2.

The diffraction angle \( \theta \) that is measured is twice the Bragg angle \( \theta_B \). The total number of scattered x rays depends on the area under this curve and will be proportional to the product of the height \( H_j \) and the width \( T_j \) for each line. It has been shown that

\[
H_j \propto \frac{1}{\sin \theta_B_j}
\]

and

\[
T_j \propto \frac{1}{\cos \theta_B_j}.
\]

The third factor which affects the total number of scattered x rays is the probability of a particular set of crystal planes being favorably oriented so as to satisfy the Bragg condition in order to scatter x rays into a narrow cone at the appropriate angle \( \theta_j \). In spite of the random orientation of the crystals, this probability is not the same for all sets of planes, and it has been shown to be proportional to \( \cos \theta_B_j \).

The probability \( I_j \) of an x ray being scattered into the \( j \)'th diffraction line is therefore proportional to these factors, i.e.,

\[
I_j \propto F_j^2(x_j, Z)(1 + \cos^2 \theta_j)p_j H_j T_j \cos \theta_B_j
\]

\[
= CF_j^2(x_j, Z)p_j \frac{1 + \cos^2 \theta_j}{\sin(\theta_j/2)},
\]

where \( C \) is a constant. Combining Eqs. (3) and (5) shows that the momentum transfer argument is related to the crystal-plane spacing by

\[
x_j = \frac{1}{\lambda_M \sin(\theta_j/2)} = \frac{\sin(\theta_j/2)}{2d_j \sin(\theta_j/2)} = \frac{1}{2d_j}.
\]

Therefore,
\[
F_j^2(x_j, Z) = F_j^2 \left( \frac{1}{2d_j} Z \right).
\]

(10)

This form highlights the fact that \( F_j \) is a property of the material only, i.e., it does not depend on wavelength or scatter angle. Since the first three factors in \( F_j \) [Eq. (8)] have no wavelength or scatter-angle dependence, it is convenient to group them into a new factor \( q_j \), where

\[
q_j = CF_j^2(x_j, Z) p_j.
\]

(11)

Combining Eqs. (8) and (11),

\[
I_j = q_j \frac{1 + \cos^2 \theta_j}{\sin(\theta_j/2)},
\]

(12)

and the total number of x rays scattered into the \( j \)th diffraction line is [combining Eqs. (4) and (12)]

\[
\nu_j = N_0 q_j \frac{1 + \cos^2 \theta_j}{\sin(\theta_j/2)}.
\]

(13)

The differential distribution of scatter per unit angle for the mono-energetic beam can be written by approximating each of these narrow peaks \( \nu_j \) as a \( \delta \)-function and summing the resulting series of \( \delta \)-functions, giving the mono-energetic cross section \( d\sigma_M(\theta)/d\theta \) as [using Eq. (13)]

\[
\frac{d\sigma_M}{d\theta}(\theta) = \alpha \sum_j \nu_j \delta(\theta - \theta_j)
\]

\[
= \alpha \sum_j N_0 q_j \frac{1 + \cos^2 \theta}{\sin(\theta/2)} \delta(\theta - \theta_j),
\]

(14)

where \( \alpha \) is a proportionality constant which gives \( d\sigma_M(\theta)/d\theta \) the units of a cross section.

**2. Poly-energetic beam**

When a poly-energetic beam of x rays is incident on the object, \( N_0 \) can be expressed as

\[
N_0 = \int dN_0/d\lambda(\lambda) d\lambda,
\]

(15)

where \( dN_0/d\lambda(\lambda) \) is the number of x rays per unit wavelength having wavelength \( \lambda \). The differential distribution of scatter per unit angle for scatter by the \( j \)th set of crystal planes can be written as \( d\nu_j(\theta)/d\theta \), where

\[
\frac{d\nu_j}{d\theta}(\theta) = \frac{dN_0}{d\theta}(\theta, d_j) I_j(\theta),
\]

(16)

and \( dN_0/d\theta(\theta, d_j)/d\theta \) is the differential distribution assuming all of the incident x rays are scattered by the \( j \)th set of crystal planes and \( I_j(\theta) \) is the angle-dependent probability of an x ray being scattered by the \( j \)th set of crystal planes. It is convenient to introduce a new variable \( S \) which is equal to \( dN_0/d\theta \), and since \( dN_0/d\theta \) can be written in terms of the x-ray spectrum \( dN_0/d\lambda \), the following relation exists:

\[
S = \frac{dN_0}{d\theta} = \frac{dN_0}{d\lambda} \frac{d\lambda}{d\theta}.
\]

(17)

From the Bragg condition [Eq. (1)],

\[
\frac{d\lambda}{d\theta} = d \cos(\theta/2).
\]

(18)

Combining Eqs. (17) and (18) and evaluating at the wavelength defined by the Bragg condition [Eq. (1)] gives

\[
S(\theta, d) = \frac{dN_0}{d\lambda}(\lambda) \frac{d \cos(\theta/2)}{\lambda - 2d \sin(\theta/2)},
\]

(19)

which describes the differential distribution of scatter per unit angle for a set of crystal planes with plane spacing \( d \), assuming that every x ray is scattered by this particular set of planes. This can be related to the mono-energetic case. The wavelength \( \lambda_M \) results in scatter at a particular angle \( \theta' \) for a plane spacing \( d \) through the Bragg condition [Eq. (1)]. Therefore,

\[
d = \frac{\lambda_M}{2 \sin(\theta'/2)},
\]

(20)

and combining Eqs. (19) and (20) gives

\[
S(\theta, \theta') = \left. \frac{dN_0}{d\lambda}(\lambda) \right|_{\lambda = \lambda_M[\sin(\theta'/2)/\sin(\theta'/2)]} \times \frac{\lambda_M}{2 \sin(\theta'/2)} \cos(\theta/2).
\]

(21)

\( S(\theta, \theta') \) is the differential distribution of scatter per unit angle for a set of crystal planes which results in scatter at the single angle \( \theta' \) in the mono-energetic case, assuming that every x ray is scattered by this set of planes. Combining Eqs. (16) and (12) yields

\[
\frac{d\nu_j}{d\theta}(\theta) = S(\theta, \theta') q_j \frac{1 + \cos^2 \theta}{\sin(\theta/2)},
\]

(22)

which gives the scatter distribution for the \( j \)th set of crystal planes, and Eq. (22) can be summed over all of the sets of crystal planes to give the polyenergetic cross section \( d\sigma_p(\theta)/d\theta \) as

\[
\frac{d\sigma_p}{d\theta}(\theta) = \alpha \sum_j \frac{d\nu_j}{d\theta}(\theta)
\]

\[
= \alpha \sum_j S(\theta, \theta') q_j \frac{1 + \cos^2 \theta}{\sin(\theta/2)},
\]

(23)

and once again the proportionality constant \( \alpha \) gives \( d\sigma_p(\theta)/d\theta \) the units of a cross section.

**3. Spectral blur as a linear superposition integral**

It is convenient to define two functions, \( g_1(\theta') \) and \( g_2(\theta, \theta') \), where [using Eq. (14)]:

\[
g_1(\theta') = \frac{\sin(\theta'/2)}{1 + \cos^2 \theta'} \frac{d\sigma_M}{d\theta}(\theta, \theta')
\]

\[
= \alpha \sum_j N_0 q_j \delta(\theta' - \theta_j),
\]

(24)

and,
and defining \( h(\theta) \) as the linear superposition of these two functions gives

\[
h(\theta) = \int_{-\infty}^{\infty} g_1(\theta')g_2(\theta, \theta') d\theta' \]
\[
= \int_{-\infty}^{\infty} \sum_j N_0 q_j \delta(\theta - \theta_j) \frac{S(\theta, \theta')}{N_0} \frac{1 + \cos^2 \theta}{\sin(\theta/2)} d\theta' \]
\[
= \sum_j q_j S(\theta, \theta_j) \frac{1 + \cos^2 \theta}{\sin(\theta/2)}. \tag{26}\]

Comparing Eqs. (23) and (26), it is evident that

\[
\frac{d\sigma_p}{d\theta'}(\theta) = \int_{-\infty}^{\infty} g_1(\theta')g_2(\theta, \theta') d\theta'. \tag{27}\]

This result shows that the poly-energetic cross-section measurement is the linear superposition of \( g_1 \) and \( g_2 \), where \( g_1 \) is the mono-energetic cross-section measurement weighted by a trigonometric factor [Eq. (24)] and \( g_2 \) is the normalized x-ray spectrum inversely weighted by the same factor [Eq. (25)]. This analysis allows the theoretical calculation of the expected scatter pattern for a polycrystalline material using an arbitrary x-ray spectrum when the mono-energetic result and the x-ray spectral shape are known. This calculated scatter pattern can then be compared with experimental results. While this result has only been rigorously proven for the case of a polycrystalline material for which the cross section consists of a series of \( \delta \)-functions, it is reasonable to assume that since the continuous cross section for an amorphous material could be represented as a sum of many \( \delta \)-functions, this linear superposition analysis should still apply.

**III. METHODS**

**A. System description**

The experimental set-up is shown schematically in Fig. 3. The scatter object is supported on a specimen stage in the path of a 1 mm x 1 mm x-ray beam (GE MaxiRAY 125 x-ray tube). The source-object distance is 25 cm. The transmitted primary beam is blocked by an approximately 3 mm thick lead blocker while low-angle scatter is detected by a CsI x-ray image intensifier (XRJ) (Thomson) placed a distance \( L = 30 \text{ cm} \) from the object. The light from the output phosphor of the XRJ is coupled through a lens to a 1024 x 1024 CCD (Tektronix TK1024). With this geometry and a tube voltage of 70 kVp, most of the coherent-scatter diffraction pattern intensity occurs within a radius of 5 cm from the primary beam \((\theta_i < 10^\circ)\).

Typically, small objects (\( W < 3 \text{ cm} \)) will be studied. This geometry has scatter-angle resolution advantages which will be discussed later. Additionally, the use of small objects allows more of the scattered x rays to penetrate the object, which in turn allows the use of lower energy x rays for which the coherent-scatter cross-section is larger.

**B. Angular resolution of measured cross sections**

Coherent-scatter cross-section measurements performed with this system take advantage of the fact that the objects studied will generally produce circularly symmetric diffraction patterns. Fig. 4 shows a single diffraction pattern acquired using the system. A set of concentric rings centered on the primary beam axis has been overlaid. The value of the cross section at angle \( \theta_i \) is calculated by integrating the signal in ring \( i \) and normalizing by the solid angle subtended at the object by ring \( i \).

There are a series of factors which result in an angular blurring of the cross sections measured with this system due to uncertainties in the scatter angle \( \theta_i \). The effect of each of these factors is discussed below.
1. Effect of object size

For an object-detector separation $L$ and a detection point a distance $R$ from the beam axis (Fig. 1), the scatter angle is given by

$$\theta = \tan^{-1}\left(\frac{R}{L}\right).$$

Therefore, the relative scatter-angle uncertainty due to the uncertainty in $L$ is given by

$$\frac{\Delta \theta_L}{\theta} = \frac{R \cos^2 \theta \Delta L / L^2}{\tan^{-1}(R/L)}.$$  \hspace{1cm} (29)

At small scatter angles, this expression can be approximated by

$$\frac{\Delta \theta_L}{\theta} = \frac{\Delta L}{L}.$$  \hspace{1cm} (30)

For an object of width $W$ located a distance $L$ from the detector (Fig. 1), the root-mean-square (RMS) uncertainty $\Delta L_{\text{RMS}}$ in the object-detector separation is given by

$$\Delta L_{\text{RMS}} = \left[\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{w}{W}\right)^2\right]^{1/2} = \left[\frac{\left(\frac{w}{W}\right)^2}{\int_{-w/2}^{w/2} W^2 \, dW} \right]^{1/2} = \left[\frac{w^2}{12} - 0\right]^{1/2}.$$  \hspace{1cm} (31)

Therefore, the relative RMS uncertainty in the scatter angle due to the object size is

$$\frac{\Delta \theta_{\text{RMS,object}}}{\theta} = \frac{\Delta L_{\text{RMS}}}{L} = \frac{W}{\sqrt{12}L}.$$ \hspace{1cm} (32)

The objects studied will have a width $W<3$ cm with an object-detector separation $L \sim 30$ cm. Therefore, the typical relative RMS uncertainty in the scatter angle due to the object size will be $\Delta \theta_{\text{RMS,object}}/\theta < 3\%$.

2. Effect of beam width

Using Eq. (28), the relative scatter-angle uncertainty due to the uncertainty in $R$ is given by

$$\frac{\Delta \theta_R}{\theta} = \frac{\cos^2 \theta \Delta R / L}{\tan^{-1}(R/L)}.$$ \hspace{1cm} (33)

At small scatter angles, this expression can be approximated by

$$\frac{\Delta \theta_R}{\theta} = \frac{\Delta R}{R}.$$ \hspace{1cm} (34)

If the width of the x-ray beam is $R_{\text{beam}}$, the RMS beam width is $\Delta R_{\text{RMS}} = R_{\text{beam}} / \sqrt{12}$ and the relative RMS uncertainty in the scatter angle due to the beam width is

$$\frac{\Delta \theta_{\text{RMS,beam}}}{\theta} = \frac{R_{\text{beam}}}{\sqrt{12}R}.$$ \hspace{1cm} (35)

The beam width is typically $R_{\text{beam}} = 1$ mm so an average value of $R = 2$ cm gives $\Delta \theta_{\text{RMS,beam}} / \theta = 1.4\%$.

3. Effect of image intensifier distortions

Due to the fact that the detected signal is integrated in annular rings to obtain the coherent-scatter cross sections, any geometrical distortions of the diffraction patterns due to the XRII can cause a blurring of the cross sections. If necessary, a correction could be made. However, it has been estimated elsewhere that the maximum radial distortion when using an XRII with an 11 cm radius input phosphor is 2.2%. Since the system described here utilizes such an XRII, but uses only a 5 cm radius area of interest, this effect is even smaller.

4. Effect of ring width

The effect due to the width of the annular rings in which the signal is integrated has the same form as the effect due to beam width [Eq. (35)]. Therefore, if the rings have width $R_{\text{ring}}$, the relative RMS uncertainty in the scatter angle due to the ring width is

$$\frac{\Delta \theta_{\text{RMS,ring}}}{\theta} = \frac{R_{\text{ring}}}{\sqrt{12}R}.$$ \hspace{1cm} (36)

The value of $R_{\text{ring}}$ is chosen to ensure that this effect is small. The ring width is $R_{\text{ring}} \sim 3$ mm so an average value of $R = 2$ cm gives a typical value of $\Delta \theta_{\text{RMS,ring}} / \theta = 4.3\%$.

5. Effect of self-attenuation

For the diffraction patterns to be unaffected by the attenuation caused by a finite object, it is necessary to assume that the attenuation of the x rays is the same along all paths through the object. In practice, there will be differences due to regions of varying density or geometry which will cause deviations from circular symmetry in the diffraction patterns. In addition, x rays scattered at larger angles will generally pass through more of the object than x rays scattered at small angles. These effects will cause artifacts in the diffraction patterns. At 70 kVp, the average x-ray linear attenuation coefficient in soft tissue is approximately 0.24 cm$^{-1}$. Therefore, for the small, fairly homogeneous objects that will be studied with this system, these artifacts will be small. For example, with a 3 cm thick rectangular homogeneous object consisting of soft tissue, the difference in attenuation between a path directly through the object and one which is deviated by an angle of $10^\circ$ is only 1%.

6. Effect of spectral width

The effect of the poly-energetic x-ray spectrum on the cross-section measurements was described in detail in sections II B 2 and II B 3 as a linear superposition of the monoenergetic cross-section measurement (scaled by a weighting
factor) with a function that is directly related to the x-ray spectral shape. The relative uncertainty in measured scatter angles due to the spectrum can be quantified as follows. From Eq. (3), the scatter angle for a particular value of the momentum transfer argument x is given by

$$\theta = 2 \sin^{-1} (\lambda x).$$

Therefore, the relative scatter-angle uncertainty due to the uncertainty in the wavelength \( \lambda \) is given by

$$\frac{\Delta \theta}{\theta} = \frac{2 \Delta \lambda / \cos(\theta/2)}{2 \sin^{-1}(\lambda x)}.$$  

(38)

At small scatter angles, this expression can be approximated by

$$\frac{\Delta \theta}{\theta} = \frac{\Delta \lambda}{\lambda}.$$  

(39)

Therefore, the relative RMS uncertainty in the scatter angle due to the spectral width is

$$\frac{\Delta \theta_{\text{RMS, spectrum}}}{\theta} = \frac{\Delta \lambda_{\text{RMS}}}{\lambda}.$$  

(40)

A 70 kVp beam with 0.30 g/cm² Gd filtration (discussed below) results in a 14% RMS spectral width which gives \( \Delta \theta_{\text{RMS, spectrum}} / \theta = 14\% \). This shows that the x-ray spectrum is the most important factor which will affect the angular resolution of the experimental cross-section measurements. The x-ray spectrum is therefore studied in detail in the following sections.

C. Improving angular resolution by spectral filtration

The angular resolution of the measured cross sections is proportional to the spectral width of the x-ray spectrum. This blurring can be reduced through the use of spectral shaping filters (K-edge filters) at the expense of beam intensity. The method of Tucker et al. was used to calculate x-ray spectra for various tube potentials. These spectra were then mathematically propagated through filter materials of various thicknesses. The beams were also filtered with 3 g/cm² of water which is representative of a typical object that will be studied. This filtering was performed by dividing the x-ray spectra into 0.5-keV wide energy bins and attenuating each energy bin by the appropriate attenuation coefficient, thus giving the spectra on the exit side of the object. The relative RMS width of each exit spectrum was calculated, as was the total x-ray fluence. The RMS width of the x-ray spectrum is defined by \( \Delta x_{\text{RMS}} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \) and the relative RMS spectral width is this quantity divided by the mean wavelength. The relative widths and signal intensities corresponding to the different spectra were compared and used to decide upon an appropriate filter. A variety of filter materials were studied with atomic numbers ranging from Z=41 through Z=82 (niobium, silver, cadmium, iodine, gadolinium, thulium, hafnium, tungsten, platinum, gold, mercury, thallium, and lead) which have K-edge energies between 19 keV and 88 keV.

D. Verification of angular resolution

The effect of the x-ray spectral width on the cross-section measurements was investigated by comparing the ideal diffraction pattern with measurements made using a poly-energetic 70 kVp beam with 0.30 g/cm² Gd filtration. It was assumed that all sources of cross-section blurring other than the poly-energetic x-ray spectrum could be neglected. Scatter from powdered aluminum using a mono-energetic x-ray beam ideally consists of eight rings of appreciable scatter intensity. The ideal cross section was calculated knowing that aluminum has a face-centered cubic structure with a lattice constant of 4.049 × 10⁻¹⁰ m. Form factor values and multiplicity factors were taken from Cullity. The theoretical poly-energetic result was calculated by performing the linear superposition described by Eqs. (24), (25), and (27). Ten single-exposure (70 kVp, 200 mAs, 0.30 g/cm² Gd filtration) diffraction patterns from a sample of powdered aluminum were then averaged to obtain an experimental coherent-scatter diffraction pattern.

E. Coherent-scatter cross sections of selected materials

Diffraction patterns were acquired with this system for several different materials using a 70 kVp x-ray beam energy, 200 mAs, and 0.30 g/cm² Gd filtration. The diffraction patterns were segmented into 16 annular rings and the signal in each ring was integrated to give the scatter intensity at the scatter angle corresponding to the mid-point of that ring. The integrated values were normalized by the solid angle subtended at the object by each ring, thereby giving scatter per unit solid angle as a function of scatter angle (related to the coherent-scatter cross section \( d\sigma(\theta)/d\Omega \) through a proportionality constant).

F. Comparison with diffractometer data

As a final test of the ability of this method to accurately measure coherent-scatter cross sections, the results for Lucite were compared with measurements performed on a diffractometer using Cu Kα radiation which has a wavelength \( \lambda = 1.54 \times 10^{-10} \) m (8.0 keV). The scatter from Lucite was measured from 5° to 60° using the diffractometer. Since the diffractometer measures scatter per unit solid angle, the data were processed by multiplying by 2\( \pi \sin \theta \) to give scatter per unit angle. Following this, Eq. (27) was used to perform a linear superposition, and then finally the data were divided by 2\( \pi \sin \theta \) to give scatter per unit solid angle once again. The result was compared with the experimental data acquired using the image-intensifier based system.

IV. RESULTS

A. Improving angular resolution by spectral filtration

The theoretical study of the effect of spectral-shaping filters on the combination of relative spectral width and signal intensity determined that a good practical choice is to use a gadolinium filter (K-edge at 50 keV) with a 70 kVp beam. Other filters can be used with different kVp values to obtain
similar spectral widths. Using gadolinium, the corresponding relative RMS spectral width is shown in Fig. 5 for different filter thicknesses. With no filter, the relative RMS width is 27%. Using a 0.30 g/cm² filter as chosen, the relative RMS width is reduced to 14% at the expense of a reduction in signal intensity to 9% (Fig. 6). The relative RMS width can be reduced to 10% with a 0.50 g/cm² filter if a reduction in signal intensity to 3% can be tolerated through the use of longer exposures or larger pixels. This system is not designed for long exposures but in principle there is no reason why these measurements can not be performed with an extremely thick filter and exposures that are as long as necessary to reach sufficient noise levels.

B. Verification of angular resolution

Figure 7 is a theoretical plot of scatter per unit angle as a function of scatter angle for polycrystalline aluminum powder in an ideal case, using mono-energetic X-rays with a wavelength $\lambda = 0.28 \times 10^{-10}$ m (44 keV) [calculated using Eq. (13)].

The resulting predicted curve of scatter vs angle for the first line (i.e., lowest scatter angle) in the powdered aluminum diffraction pattern evaluated using Eq. (22) and the spectrum in Fig. 8 is shown in Fig. 9. Comparison of Figs. 8 and 9 illustrates that the shape of the superposition kernel is very similar to the shape of the X-ray spectrum as expected.
Fig. 9. Theoretical curve of scatter per unit angle as a function of scatter angle for the first line in the polycrystalline aluminum powder diffraction pattern using the poly-energetic x-ray spectrum [calculated using Eq. (22)].

since Eq. (22) indicates that the kernel is the spectrum modified by a trigonometric factor. The result of calculating the entire diffraction pattern using Eqs. (22) and (23) is compared in Fig. 10 with the experimental curve of scatter per unit angle for powdered aluminum. The theoretical curve shows only single-coherent scatter with the energy spectrum being the only source of cross-section broadening. The horizontal error bars indicate the residual scatter angle uncertainty introduced by the other factors described in sections III B 1 through III B 5. The experimental curve is obtained by integrating in circular annuli the signal from diffraction patterns for a powdered aluminum sample. The similarity between the two curves suggests that indeed the energy spectrum is the most significant factor blurring the experimental cross sections, although additional blurring of a few percent is observed at higher angles, possibly due to the other factors discussed. Additionally, since the theoretical curve does not include multiple scatter and Compton scatter, this comparison indicates that there is very little contamination from either of these sources.

C. Coherent-scatter cross sections of selected materials

Diffraction patterns obtained with this system using single exposures (70 kVp, 252 mAs, 0.30 g/cm$^2$ Gd filtration) are shown in Fig. 11. With the exception of the tungsten foil pattern, all of the diffraction patterns exhibit circular symmetry. The bright spots in the tungsten pattern are indicative of long range structure in the tungsten foil. Conversely, the well-defined rings in the aluminum powder pattern are typical of a polycrystalline material where a random ensemble of crystal orientations average to give circular rings, while amorphous materials like Lucite or water give a much broader, blurred-out diffraction pattern. Note that all of these patterns would appear sharper if mono-energetic x rays were used. For example, in an ideal case the diffraction pattern for aluminum powder would be a series of $\delta$-function rings, but due to the processes discussed previously, these $\delta$-functions are blurred into narrow rings. An important observation to make is that the diffraction patterns of pure hydroxyapatite and a human tooth are very similar. This is expected since hydroxyapatite makes up a large proportion of the mass of teeth.

The coherent-scatter cross-section measurements for water, Lucite, hydroxyapatite, and the human tooth measured from these diffraction pattern images are shown in Fig. 12. This figure shows that these materials can be easily distinguished by cross-section measurements made with this system. Also, the similarity between pure hydroxyapatite and the human tooth is confirmed (within a constant scaling factor).

It is difficult to compare these measurements with previous literature due to the fact that diffraction measurements
are usually performed with mono-energetic x rays. The angle of peak scatter for each case can however be compared with the data of Kosanetzky et al. which were acquired using mono-energetic x rays with a wavelength \( \lambda = 1.79 \times 10^{-10} \) m (6.935 keV). Note that their data are presented as a function of the momentum transfer argument \( x \) whereas the cross sections measured here are presented as a function of scatter angle. From Fig. 12, Lucite has an angle of peak scatter at about 2.5°. Since this was measured with a spectrum with a mean wavelength \( \lambda = 0.28 \times 10^{-10} \) m (44 keV), this corresponds to \( x = 0.08 \times 10^{10} \) m\(^{-1}\). The cross sections of water and hydroxyapatite both peak at about 5°, which corresponds to \( x = 0.16 \times 10^{10} \) m\(^{-1}\). These agree very well with the data presented by Kosanetzky et al. for Lucite, water, and bone. Also, qualitatively the shapes of the cross sections measured here agree with their data. Bradley et al. have also measured the cross sections of Lucite and water using \( \gamma \) rays from \(^{241}\)Am which have a wavelength \( \lambda = 0.208 \times 10^{-10} \) m (59.54 keV). They recorded peak scatter for Lucite and water at 2° and 4°, respectively, which correspond to \( x = 0.08 \times 10^{-10} \) m\(^{-1}\) and \( x = 0.17 \times 10^{-10} \) m, respectively. Note that the resolution of their measurements was 1°. These three data sets are tabulated in Table I. Harding et al. have also published cross sections of Lucite and water measured with a continuous 120 kVp spectrum. These cross sections are qualitatively very similar to the cross sections presented here. Of course they peak at a smaller angle due to the higher energy used in their experiments.

D. Comparison with diffractometer data

The raw diffractometer data for Lucite are shown in Fig. 13. After processing, these data are compared with the image-intensifier data in Fig. 14. There is excellent agreement between the two curves, indicating that it is possible to relate mono-energetic data and poly-energetic data through the linear superposition integral.

**TABLE I. Values of the momentum transfer argument for peak scatter from Lucite, water, and hydroxyapatite from the present study compared with values from two previously published studies.**

<table>
<thead>
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<th>Westmore, Fenster, and Cunningham: Angular-dependent coherent scatter</th>
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<tbody>
<tr>
<td><strong>Momentum transfer argument for peak scatter</strong> ( x ) ( \times 10^{10} ) m(^{-1})</td>
<td><strong>Present study</strong></td>
</tr>
<tr>
<td>Lucite</td>
<td>0.08</td>
</tr>
<tr>
<td>Water</td>
<td>0.16</td>
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<tr>
<td>Hydroxyapatite</td>
<td>0.16</td>
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![Fig. 12. Measured curves of scatter per unit solid angle as a function of scatter angle for water, Lucite, hydroxyapatite, and the human tooth.](image1)

![Fig. 13. Measured curve of scatter per unit solid angle performed using a diffractometer with Cu Kα radiation which has a wavelength \( \lambda = 1.54 \times 10^{-10} \) m (8.0 keV).](image2)

![Fig. 14. Result of performing a linear superposition of the diffractometer data with the poly-energetic x-ray spectrum compared with the curve of scatter per unit solid angle actually measured using this spectrum.](image3)
V. CONCLUSIONS

A method of measuring the differential coherent-scatter cross section using an x-ray image intensifier-based system has been described. The relative angular spread in the cross-section measurements introduced by the system is approximately 15%. This spread is determined largely by the polyenergetic x-ray spectrum employed and not by geometrical factors or intensifier distortions.

The cross-section measurements presented in this paper show that in spite of the polyenergetic spectrum employed, there are still significant differences between the materials examined. These distinctions allow these materials to be identified. Composite materials such as bone samples are expected to give diffraction patterns which will be a summation of their constituent parts. If the cross sections of these materials are distinctly different, it may be possible to quantify the amount of each material present. The hydroxyapatite in bone has a distinctive cross section and it may be possible to quantify the amount of bone mineral present using this method, independently of other tissue constituents and density.

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